KD EDUCATION ACADEMY [9582701166]

Time : 7 Hour | STD 11 Science Physics | Total Marks : 310

kd 90+ ch- 7 Gravitation

*** Choose The Right Answer From The Given Options.[1 Marks Each] [55]**

1. Law of areas is valid only when gravitational force is:
   (A) Conservative force.
   (B) Central force.
   (C) Attractive force.
   (D) Weak force.

   Ans. :
   b. Central force.

2. A planet has twice the density of earth but the acceleration due to gravity on its surface is exactly the same as on the surface of earth. It radius in terms of radius of earth R will be:
   (A) $\frac{R}{4}$
   (B) $\frac{R}{2}$
   (C) $\frac{R}{3}$
   (D) $\frac{R}{8}$

   Ans. :
   b. $\frac{R}{2}$

3. According to Kepler's law of planetary motion, if T represents time period and r is orbital radius, then for two planets these are related as:
   (A) $\left(\frac{T_1}{T_2}\right)^3 = \left(\frac{r_1}{r_2}\right)^2$
   (B) $\left(\frac{T_1}{T_2}\right)^{\frac{3}{2}} = \frac{r_1}{r_2}$
   (C) $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$
   (D) $\left(\frac{T_1}{T_2}\right) = \left(\frac{r_1}{r_2}\right)^{\frac{2}{3}}$

   Ans. :
   c. $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

4. The escape velocity or earth is V. If the mass of a certain planet is 3 times and radius 3 times than that of the earth, then the escape velocity from the planet will be:
   (A) $3V$
   (B) $6V_e$
   (C) $\sqrt{3}V_e$
   (D) V_e

   Ans. :
   d. V_e

   Explanation:

   $$V_e = \sqrt{\frac{2GM}{R}}$$

   $$\text{or } V_e = \sqrt{\frac{2GM}{\frac{R}{4}}} = 11.2\text{km/s}.$$

5. In case of earth:
   (A) Potential is minimum at the centre.
   (B) Potential is zero, both at centre and infinity.
   (C) Field is zero both at centre and infinity.

(D) Potential is same, both at centre and infinity but no zero.

Ans. :

a. Potential is minimum at the centre.

c. Field is zero both at centre and infinity.

6. If the law of gravitation, instead of being inversesquare law, becomes an inversecube law.

(A) Planets will not have elliptic orbits.

(B) Circular orbits of planets is not possible.

(C) Projectile motion of a stone thrown by hand on the surface of the earth will be approximately parabolic.

(D) There will be no gravitational force inside a spherical shell of uniform density.

Ans. :

a. Planets will not have elliptic orbits.

b. Circular orbits of planets is not possible.

c. Projectile motion of a stone thrown by hand on the surface of the earth will be approximately parabolic.

Explanation:

If the law of gravitation becomes an inverse cube law instead of inverse square law, then for a planet of mass m revolving around the sun of mass M, we can write

$F = \frac{GMm}{R^3} = \frac{mv^2}{R}$ (where R is the radius of orbiting planet)

$\Rightarrow$ Orbital speed $= \frac{\sqrt{GM}}{R} \Rightarrow V \propto \frac{1}{R}$

Time period of revolution of a planet

$T = \frac{2\pi R}{v} = \frac{2\pi R}{\frac{\sqrt{GM}}{R}} = \frac{2\pi R^2}{\sqrt{GM}}$

$\Rightarrow T^2 \propto R^4$

Hence, orbit will not be elliptical.

[For elliptical orbit $T^2 \propto R^3$]

The circular orbits of the planets is not possible according to new law of gravitation.

As force $F = \left(\frac{GM}{R^3}\right)m$ = g'm

where, g' $= \frac{GM}{R^3}$

As g', acceleration due to gravity is constant, hence path followed by a projectile will be approximately parabolic, $(\text{as } T \propto R^2)$.

Also, gravitational force inside a spherical shell of uniform density will have some value. So, only option (d) is incorrect.

7. The change in potential energy, when a body of mass m is raised to a height nR from earth's surface is (R = radius of earth):

(A) $mgR\left(\frac{n}{n-1}\right)$ (B) $nmgR$ (C) $mgR\left(\frac{n^2}{n^2+1}\right)$ (D) $mgR\left(\frac{n}{n+1}\right)$

Ans. :

d. $mgR\left(\frac{n}{n+1}\right)$

Explanation:

Change in potential energy,

$$\Delta E = GMm\left(\frac{1}{R} - \frac{1}{R+nR}\right)$$
$$= \frac{GMm}{R(R+nR)} \times nR$$
$$= \frac{GMm}{R}\left(\frac{n}{1+n}\right)$$
$$= gRm\left(\frac{n}{1+n}\right)$$

8. If two satellites of different masses are revolving in the same orbit, they have same:

(A) Speed. (B) Energy.

(C) Time period. (D) Angular momentum

Ans. :

a. Speed.

c. Time period.

Explanation:

The speed and time period of revolution of a satellite is independent of mass of the satellite but energy and angular momentum of a satellite depend upon mass of the body.

9. Three particles each of mass m are kept at vertices of an equilateral triangle of side L. Th gravitational potential energy possessed by the system is:

(A) $\frac{-Gm^2}{L}$ (B) $\frac{-3Gm^2}{L}$ (C) $-\frac{2Gm^2}{L}$ (D) $\frac{+3Gm^2}{L}$

Ans. :

b. $\frac{-3Gm^2}{L}$

10. If the mass of the earth is doubled and its radius halved, then new acceleration due to the gravity g' is:

(A) g' = 4g (B) g' = 8g (C) g' = g (D) g' = 16g

Ans. :

b. g' = 8g

11. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is:

(A) gx (B) $\frac{gR}{R-x}$ (C) $\frac{gR^2}{R+x}$ (D) $\left(\frac{gR^2}{R+x}\right)^{\frac{1}{2}}$

Ans. :

d. $\left(\frac{gR^2}{R+x}\right)^{\frac{1}{2}}$

12. Escape velocity of a planet is v. If radius of the planet remains same and mass becomes 4 times, the escape velocity becomes:

(A) $4v_e$ (B) $2v_e$ (C) v_e (D) $\frac{v_e}{2}$

Ans. :

b. $2v_e$

Explanation:

Escape velocity,

$v_e = \sqrt{2gR}$

$= \sqrt{\frac{2GM}{R}}, v_e \propto \sqrt{M}.$

13. The time period of a second's pendulum in a satellite is:

(A) Zero. (B) 2

(C) Infinity. (D) Depends on mass of body.

Ans. :

c. Infinity.

Explanation:

Inside the satellite the effective value of g = 0, so time period $T = 2\pi\sqrt{\frac{l}{g}}$ is infinity.

14. The ratio of the magnitude of potential energy and kinetic energy of a satellite is:

(A) 1 : 2 (B) 2 : 1 (C) 3 : 1 (D) 1 : 3

Ans. :

b. 2 : 1

15. If the gravitational potential energy at infinity is assumed to be zero, the potential energy at distance (R_e + h) from the centre of the earth:

(A) $PE = \frac{GmM_e}{(R_e+h)}$ (B) $PE = \frac{-GmM_e}{(R_e+h)}$

(C) $PE = mgh$ (D) $PE = \frac{-GmM_e}{2(R_e+h)}$

Ans. :

b. $PE = \frac{-GmM_e}{(R_e+h)}$

16. An artificial earth satellite of mass m is circling round the earth in an orbit of radius R. If the mass of the earth is M, then the total energy of the satellite is:

(A) $\frac{3GMm}{2R}$ (B) $\frac{-GMm}{2R}$ (C) $\frac{GMm}{R}$ (D) $\frac{-GMm}{R}$

Ans. :

b. $\frac{-GMm}{2R}$

Explanation:

Total energy = P.E. + K.E.

$= \frac{GMm}{R} + \frac{1}{2}mv^2$

$= -\frac{GMm}{R} + \frac{1}{2}m\frac{GM}{R}$

17. Both earth and moon are subject to the gravitational force of the sun. As observed from the sun, the orbit of the moon.

(A) Will be elliptical.

(B) Will not be strictly elliptical because the total gravitational force on it is not central.

(C) Is not elliptical but will necessarily be a closed curve.

(D) Deviates considerably from being elliptical due to influence of planets other than earth.

Ans. :

b. Will not be strictly elliptical because the total gravitational force on it is not central.

Explanation:

Moon revolves around the earth in a nearly circular orbit. When it is observed from the sun, two types of forces are acting on the moon one is due to gravitational attraction between the sun and the moon and the other is due to gravitational attraction between the earth and the moon. So moon is moving under the combined gravitational pull acting on it due to the earth and the sun. Hence, total force on the moon is not central.

18. The largest and shortest distance of earth from the sun are r_1 and r_2. Its distance from the sun when it is perpendicular to the major axis of the orbit drawn from the sun is:

(A) $\frac{r_1+r_2}{4}$ (B) $\frac{r_1+r_2}{r_1-r_2}$ (C) $\frac{2r_1r_2}{r_1+r_2}$ (D) $\frac{r_1+r_2}{2}$

Ans. :

c. $\frac{2r_1r_2}{r_1+r_2}$

19. If M is the mass of the earth and R its radius, the ratio of the gravitational acceleration and the gravitational constant is:

(A) $\frac{R^2}{M}$ (B) $\frac{M}{R^2}$ (C) MR^2 (D) $\frac{M}{R}$

Ans. :

b. $\frac{M}{R^2}$

20. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the earth's surface to a height equal to the radius R of the earth, is:

(A) $\frac{1}{2}m\,g\,R$ (B) $2m\,g\,R$ (C) $m\,g\,R$ (D) $\frac{1}{4}m\,g\,R$

Ans. :

a. $\frac{1}{2}m\,g\,R$

21. What is the weight of a 700gm of body on a planet whose mass is $\frac{1}{7}\text{th}$ of that of earth and radius is $\frac{1}{2}$ of earth:

(A) 400gm (B) 300gm (C) 700gm (D) 500gm

Ans. :

a. 400gm

Explanation:

On, earth, $g = \frac{GM}{R^2}$

On planet, $g' = \frac{\frac{GM}{7}}{\left(\frac{R}{2}\right)^2} = \frac{4}{7}g$

$\therefore$ Weight of body on planet,

$= \mathrm{mg'} = 700 \times \frac{4}{7} \times \mathrm{g} = 400\mathrm{mg\ wt.}$

22. Mars has about $\frac{1}{10}\mathrm{th}$ as much mass as the earth and half as great a diameter. The acceleration of a falling body on Mars is about:

(A) $9.8ms^{-2}$ (B) $1.96ms^{-2}$ (C) $3.92ms^{-2}$ (D) $4.9ms^{-2}$

Ans. :

c. $3.92ms^{-2}$

23. A particle of mass m is at the surface of the earth of radius R. It is lifted to a heighth abov the surface of the earth. The gain in gravitational potential energy of the particle is:

(A) $\frac{\mathrm{mgh}}{\left(1-\frac{\mathrm{h}}{\mathrm{R}}\right)}$ (B) $\frac{\mathrm{mgh}}{\left(1+\frac{\mathrm{h}}{\mathrm{R}}\right)}$

(C) $\frac{\mathrm{mghR}}{(\mathrm{R+h})}$ (D) $\mathrm{Both\ (b)\ and\ (c)}$

Ans. :

d. $\mathrm{Both\ (b)\ and\ (c)}$

24. The escape speed from the surface of earth is v_e. The escape speed from the surface of a planet whose mass and radius are 3 times those of the earth will be:

(A) v_e (B) $3v_e$ (C) $9v_e$ (D) $27v_e$

Ans. :

a. v_e

Explanation:

$$\mathrm{v_e} = \sqrt{\mathrm{vgR}}$$

$$= \sqrt{\frac{2\mathrm{GM_p}}{\mathrm{R_p}}} = \sqrt{\frac{2\mathrm{G}\times 3\mathrm{M}}{2\mathrm{R}}}$$

$$= \sqrt{\frac{2\mathrm{GM}}{\mathrm{R}}} = \mathrm{v_e}.$$

25. A satellite is orbiting just above the surface of a planet of average density ρ with period T. If G is the universal gravitational constant, the quantity $\mathrm{T}^2\rho$ is equal to:

(A) $4\frac{\pi^2}{\mathrm{G}}$ (B) $4\frac{\pi^2}{\mathrm{G}}$ (C) $4\frac{\pi}{\mathrm{G}}$ (D) $\frac{1}{\mathrm{G}}$

Ans. :

c. $4\frac{\pi}{\mathrm{G}}$

Explanation:

$$\mathrm{v} = \sqrt{\mathrm{gR}} \text{ and } \mathrm{T} = \frac{2\pi\mathrm{R}}{\mathrm{v}} = \frac{2\pi\mathrm{R}}{\sqrt{\mathrm{gR}}}$$

$$\text{or } \mathrm{T} = \frac{2\pi\mathrm{R}}{\sqrt{\frac{\mathrm{GM}}{\mathrm{R}}}} = 2\pi$$

$$\sqrt{\frac{\mathrm{R}^3}{\mathrm{G}\frac{4}{3}\pi\mathrm{R}^3\rho}}$$

$$\mathrm{T}^2\rho = 4\pi^2 \times \frac{3}{4\mathrm{G}\pi} = \frac{3\pi}{\mathrm{G}}$$

26. If the radius of earth were to increase by 1%, its mass remaining the same, the acceleration due to gravity on the surface of earth will:

(A) Increase by 1% (B) Decrease by 2% (C) Decrease by 1% (D) Increase by 2%

Ans. :

b. Decrease by 2%

Explanation:

$g = \frac{GM}{R^2}, \frac{\Delta g}{g}$

$= -2\frac{\Delta R}{R}$

$\therefore$ Percentage change in the value of g

$= \frac{\Delta g}{g} \times 100 = -2\frac{\Delta R}{R} \times 100$

$= -2(1\%) = -2\%$

27. The radii of circular orbits of two satellites around the earth are in the ratio 1 : 4, then ratio of their respective periods of revolution is:

(A) 1 : 4 (B) 4 : 1 (C) 1 : 8 (D) 8 : 1

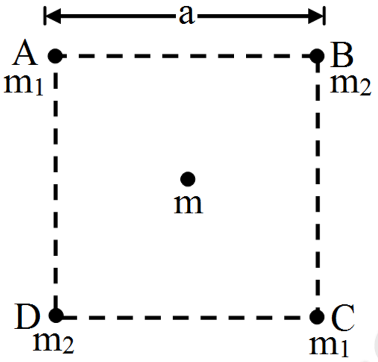
Ans. :

c. 1 : 8

Explanation:

$T^2 \propto R^3$

28. A point mass m is placed at the centre of the square ABCD of side a units as shown



below. The resultant gravitational force on mass m due to masses m_1 and m_2 plant on the vertices of square is:

(A) $\frac{Gm_1m_2}{(a\sqrt{2})^2}$

(B) $\frac{2Gm(m_1+m_2)}{a^2}$

(C) zero

(D) $\frac{Gm(m_1+m_2)}{(a\sqrt{2})^2}$

Ans. :

c. zero

29. A satellite is launched into a circular orbit of radius R around the earth. A second satellite launched into an orbit of radius 1.01R. The time period of the second satellite is larger than that of the first one by approximately:

(A) 0.5% (B) 1.5% (C) 1% (D) 3.0%

Ans. :

b. 1.5%

30. If three uniform spheres, each having mass M and radius r, are kept in such a way that each touches the other two, the magnitude of the gravitational force on any sphere due to the other two is:

(A) $\frac{GM^2}{4r^2}$ (B) $\frac{2GM^2}{r^2}$ (C) $\frac{2GM^2}{4r^2}$ (D) $\frac{\sqrt{3}GM^2}{4r^2}$

Ans. :

d. $\frac{\sqrt{3}GM^2}{4r^2}$

31. The law of areas can be interpreted as:

(A) $\frac{\Delta A}{\Delta t} = \text{constant}$ (B) $\frac{\Delta A}{\Delta t} = \frac{L}{m}$

(C) $\frac{\Delta A}{\Delta t} = \frac{1}{2}(r \times P)$ (D) $\frac{\Delta A}{\Delta t} = \frac{2L}{m}$

Ans. :

a. $\frac{\Delta A}{\Delta t} = \text{constant}$

32. Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with liquid of specific gravity 3. The gravitational force will now be:

(A) $3F$ (B) F (C) $\frac{F}{3}$ (D) $\frac{F}{3}$

Ans. :

b. F

Explanation:

The gravitational force between two bodies is independent of the presence of other bodies.

33. The radius of the orbit of a satellite is r and its kinetic energy is K. If the radius of the orbit is doubled, then the new kinetic energy K' is:

(A) 2 (B) $\frac{K}{2}$ (C) 4K (D) Data insufficient.

Ans. :

b. $\frac{K}{2}$

34. A body of mass m is placed on earth surface which is taken from earth surface to a height of h = 3R, then change in gravitational potential energy is:

(A) $\frac{1}{4}mgR$ (B) $\frac{2}{3}mgR$ (C) $\frac{3}{4}mgR$ (D) $\frac{1}{3}mgR$

Ans. :

c. $\frac{3}{4}mgR$

Explanation:

Change in gravitational potential energy

$=\frac{-GMm}{(3R+R)} - \left(\frac{-GMm}{R}\right)$

$= \frac{3}{4}\frac{GMm}{R} = \frac{3}{4}mgR.$

35. Three uniform spheres of mass M and radius R each are kept in such a way that each touches the other two. The magnitude of the gravitational force on any of the spheres due to the other two is:

(A) $\frac{\sqrt{3}}{4}\frac{GM^2}{R^2}$ (B) $\frac{3}{2}\frac{GM^2}{R^2}$ (C) $\frac{\sqrt{3}GM^2}{R^2}$ (D) $\frac{\sqrt{3}}{2}\frac{GM^2}{R^2}$

Ans. :

a. $\frac{\sqrt{3}}{4}\frac{GM^2}{R^2}$

36. Earth is flattened at the poles and bulges at the equator. This is due to the fact that:

(A) The earth revolves around the sun in an elliptical orbit.

(B) The angular velocity of spinning about its axis is more at the equator.

(C) The centrifugal force is more at the equator than at poles.

(D) None of the above.

Ans. :

c. The centrifugal force is more at the equator than at poles.

Explanation:

Higher centrifugal force causes bulging of earth at equator.

37. The period of revolution of a certain planet in an orbit of radius R is T. Its period of revolution in an orbit of radius 4R will be:

(A) $2T$ (B) $\frac{2}{2T}$ (C) $4T$ (D) $8T$

Ans. :

d. $8T$

Explanation:

$$\frac{T_2^2}{T_1^2}=\frac{r_2^3}{r_1^2}$$

$$\text{or } T_2=T_1\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$$

$$=T\left(\frac{4R}{R}\right)^{\frac{3}{2}}=8T$$

38. For a satellite in elliptical orbit which of the following quantities does not remain constant?

(A) Angular momentum. (B) Linear momentum. (C) Areal velocity. (D) All of the above.

Ans. :

b. Linear momentum.

Explanation:

In elliptical orbit, velocity keeps on changing both in magnitude and direction. Therefore, momentum does not remain constant (P = mv).

39. Two point masses m_1 and m_2 are separated by a distance r. The gravitational potential energy of the system is G_1. When the separation between the particles is doubled, the gravitational potential energy is G_2. Then, the ratio of $\frac{G_1}{G_2}$ is:

(A) 1 (B) 2 (C) 3 (D) 4

Ans. :

b. 2

40. A satellite is orbiting the earth. If its distance from the earth is increased, its:

(A) Angular velocity would increase.
(B) Linear velocity would increase.
(C) Angular velocity would decrease.
(D) Time period would increase.

Ans. :

c. Angular velocity would decrease.
d. Time period would increase.

41. The orbital speed of an artificial satellite in a circular orbit just above the earth's surface is v. For a satellite orbiting at an altitude of half the earth's radius the orbital speed is:

(A) $\left(\frac{3}{2}\right)v$
(B) $\left(\sqrt{\frac{3}{2}}\right)v$
(C) $\left(\sqrt{\frac{2}{3}}\right)v$
(D) $\left(\frac{2}{3}\right)v$

Ans. :

c. $\left(\sqrt{\frac{2}{3}}\right)v$

Explanation:

Orbital velocity close to earth, $v = \sqrt{gR}$ Orbital velocity at height $\frac{R}{2}$ will be

$$v = \sqrt{\frac{gR^2}{R+\frac{R}{2}}}$$

$$= \sqrt{\frac{2}{3}gR} = \left(\sqrt{\frac{2}{3}}\right)v$$

42. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of earth to a height equal to the radius R of the earth is:

(A) $\frac{I}{2}mgR$
(B) $2mgR$
(C) mgR
(D) $\frac{1}{4}mgR$

Ans. :

a. $\frac{I}{2}mgR$

Explanation:

Work done in raising the body

$$= \int_{R}^{2R} \frac{GMm}{x^2}dx$$

$$= \int_{R}^{2R} \frac{gR^2}{x^2}mdx$$

$$= \frac{1}{2}mgR$$

43. If the mass of sun were ten times smaller and gravitational constant G were ten times larger in magnitudes.

(A) Walking on ground would became more difficult.
(B) The acceleration due to gravity on earth will not change.
(C) Raindrops will fall much faster.
(D) Airplanes will have to travel much faster.

Ans. :

a. Walking on ground would became more difficult.
c. Raindrops will fall much faster.
d. Airplanes will have to travel much faster.

Explanation:

If the gravitational constant G becomes 10 times larger in magnitude.

$\mathrm{G}' = 10\mathrm{G}$

Gravitational field due to the earth

$\mathrm{g}' = \frac{\mathrm{G}'\mathrm{M_e}}{\mathrm{r}^2}$

$= \frac{10\mathrm{GM_e}}{\mathrm{r}^2} = 10\mathrm{g}$

Weight of a person $= \mathrm{mg}' = \mathrm{m} \times 10\mathrm{g} = 10\mathrm{mg}$

Force on the man due to sun, $\mathrm{F} = \frac{\mathrm{GM_s'm}}{\mathrm{r}^2}$

Mass of the sun $\mathrm{M_s'} = \frac{1}{10}\mathrm{M_s} \Rightarrow 10\mathrm{M_s'} = \mathrm{M_s}$

$\mathrm{F} = \frac{\mathrm{GM_s m}}{10\mathrm{r}^2}$

Weight of person becomes 10 times larger so it will be more difficult to walk. Option (a) is correct.

As g’ = 10g, the acceleration due to gravity changes. Option (b) is incorrect.

The terminal velocity $\mathrm{v_T} \propto \mathrm{g}$ and g', g' = 10g, the terminal velocity increases 10 times.

Hence the rain drops falls 10 times faster. Option (c) is correct.

As the g’ = 10g, to overcome the increase gravitational force and in order to maintain the speed the aeroplane will have to travel much faster. Option (d) is correct.

44. The gravitational potential at a plane varies inversely proportional to $\mathrm{x}^2 \left(\mathrm{i.e.,\ V} = \frac{\mathrm{k}}{\mathrm{x_2}}\right)$ then gravitational field intensity at the place is:

(A) $\frac{-\mathrm{k}}{\mathrm{x}}$ (B) $\frac{\mathrm{k}}{\mathrm{x}}$ (C) $\frac{-2\mathrm{k}}{\mathrm{x}^3}$ (D) $\frac{2\mathrm{k}}{\mathrm{x}^3}$

Ans. :

a. $\frac{-\mathrm{k}}{\mathrm{x}}$

Explanation:

Gravitational intensity,

$\mathrm{I} = -\frac{\mathrm{dV}}{\mathrm{dx}} = -\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{\mathrm{k}}{\mathrm{x}^2}\right) = \frac{-\mathrm{k}}{\mathrm{x}}$

45. Two satellites of masses m_1 and m_2 ($m_1 > m_2$) are revolving round the earth in circular orbits of radii r_1 and r_2($r_1 > r_2$) respectively. Which of the following statements is true regarding their speeds v_1 and v_2.

(A) $\mathrm{v_1} > \mathrm{v_2}$ (B) $\mathrm{v_1} < \mathrm{v_2}$ (C) $\mathrm{v_1} = \mathrm{v_2}$ (D) $\frac{\mathrm{v_1}}{\mathrm{r_1}} = \frac{\mathrm{v_2}}{\mathrm{r_2}}$

Ans. :

b. $\mathrm{v_1} < \mathrm{v_2}$

Explanation:

For satellite, orbital speed, $\mathrm{v} \propto \frac{1}{\mathrm{r}}$

Therefore, $\frac{v_1}{v_2}=\sqrt{\frac{r_2}{r_1}}<1$ or $v_1<v_2$.

46. The orbital velocity of a satellite orbiting near the surface of the earth is given by:

(A) $v\sqrt{gR_e}$, where $g=\frac{GM_e}{R_e^2}$

(B) $v=\sqrt{gR_e}$, where $g=\frac{GM_e}{R_e}$

(C) $v=\sqrt{\frac{gh}{R_e}}$, where $g=\frac{GM_e}{R_e^2}$

(D) $v=\sqrt{gh}$, where $g=\frac{GM_e}{R_e}$

Ans. :

a. $v\sqrt{gR_e}$, where $g=\frac{GM_e}{R_e^2}$

Explanation:

Orbital velocity of satellite, $v=\sqrt{\frac{GM_e}{(R_e+h)}}$

If the satellite is close to the surface of the earth, h = 0

$\Rightarrow v=\sqrt{\frac{GM_e}{R_e}}$

$\Rightarrow v=\sqrt{\left(\frac{GM_e}{R_e^2}\right)R_e}$

$\sqrt{gR_e},\left[\because g=\frac{GM_e}{R_e^2}\right]$

47. A particle is kept at rest at a distance R_e (earth's radius) above the earth's surface. The minimum speed with which it should be projected so that it does not return is (mass of earth = M_e)

(A) $\sqrt{\frac{6M_e}{4R_e}}$ (B) $\sqrt{\frac{GM_e}{2R_e}}$ (C) $\sqrt{\frac{GM_e}{R_e}}$ (D) $\sqrt{\frac{2GM_e}{R_e}}$

Ans. :

c. $\sqrt{\frac{GM_e}{R_e}}$

48. If the acceleration due to gravity at earth is 'g' and mass of earth is 80 times that of moon and radius of earth is 4 times that of moon, the value of 'g' at the surface of moon will be:

(A) g (B) $\frac{g}{20}$ (C) $\frac{g}{5}$ (D) $\frac{320}{g}$

Ans. :

c. $\frac{g}{5}$

Explanation:

Let M and R be the mass and radius of earth M' and R' are mass and radius of moon. Then $R'=\frac{R}{4}$ and $M'=\frac{M}{80}$ Let g and g' the acceleration due to gravity on the surface of earth and moon respectively. Then $g=\frac{GM}{R^2}$

and $g'=\frac{GM'}{R'^2}=G\frac{\frac{M}{80}}{\left(\frac{R}{4}\right)^2}$

$=\frac{1}{5}\frac{GM}{R^2}=\frac{1}{5}g.$

49. If both the mass and the radius of the earth decreases by 1%

(A) The escape velocity would increase.

(B) The acceleration due to gravity would increases.

(C) The escape velocity would decrease.

(D) The acceleration due to gravity would decrease.

Ans. :

b. The acceleration due to gravity would increases.

Explanation:

Escape velocity, $\mathrm{v_e} = \sqrt{\frac{2\mathrm{GM}}{\mathrm{R}}}$

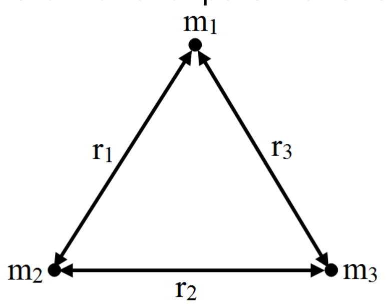
$\mathrm{v_e'} = \sqrt{\frac{2\mathrm{G}\left(\frac{99\mathrm{M}}{100}\right)}{\left(\frac{99\mathrm{R}}{100}\right)}}$

$= \sqrt{\frac{2\mathrm{GM}}{\mathrm{R}}} = \mathrm{v_e}$

Also, $\mathrm{g} = \frac{\mathrm{GM}}{\mathrm{R}^2}$

and $\mathrm{g'} = \frac{\mathrm{G}\left(\frac{99\mathrm{M}}{100}\right)}{\left(\frac{99\mathrm{R}}{100}\right)} = \frac{100}{99}\mathrm{g}.$

50. Gravitational potential energy of a system of particles as shown in the figure is:



(A) $\frac{\mathrm{Gm_1m_1}}{\mathrm{r_1}} + \frac{\mathrm{Gm_2m_3}}{\mathrm{r_3}} + \frac{\mathrm{Gm_1m_3}}{\mathrm{r_3}}$

(B) $\left(\frac{-\mathrm{Gm_1m_2}}{\mathrm{r_1}}\right) + \left(\frac{-\mathrm{Gm_2m_3}}{\mathrm{r_2}}\right) + \left(\frac{-\mathrm{Gm_1m_3}}{\mathrm{r_3}}\right)$

(C) $\frac{-\mathrm{Gm_1m_2}}{\mathrm{r_1}} - \frac{\mathrm{Gm_2m_3}}{\mathrm{r_2}} + \frac{\mathrm{Gm_1m_3}}{\mathrm{r_3}}$

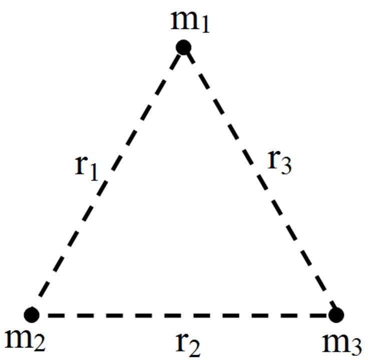
(D) $\frac{\mathrm{Gm_1m_2}}{\mathrm{r_1}} + \frac{\mathrm{Gm_2m_3}}{\mathrm{r_2}} - \frac{\mathrm{Gm_1m}}{\mathrm{r_3}}$

Ans. :

b. $\left(\frac{-\mathrm{Gm_1m_2}}{\mathrm{r_1}}\right) + \left(\frac{-\mathrm{Gm_2m_3}}{\mathrm{r_2}}\right) + \left(\frac{-\mathrm{Gm_1m_3}}{\mathrm{r_3}}\right)$

Explanation:

For a system of particles, all possible pairs are taken and total gravitational potential energy is the algebraic sum of the potential energies due to each pair, applying the principle of superposition. Total gravitational potential energy.



$$= \frac{-Gm_1m_2}{r} - \frac{Gm_2m_3}{r_2} - \frac{Gm_1m_3}{r_3}$$

$$= \left(\frac{-Gm_1m_2}{r_1}\right) + \left(\frac{-Gm_2m_3}{r_2}\right) + \left(\frac{-Gm_1M_3}{r_3}\right)$$

51. The escape velocity of a body from the earth is ve. If the radius of earth contracts to $\frac{1}{4}\text{th}$ of its value, keeping the mass of the earth constant, escape velocity will be:

(A) Doubled. (B) Halved. (C) Tripled. (D) Unaltered.

Ans. :

a. Doubled.

Explanation:

Give, escape speed $v_e = \sqrt{\frac{2GM_e}{R_e}}$

Where, M_e = mass of the earth, R_e = radius of the earth,

Now, radius of earth $= R' = \frac{R_e}{4}$

$$\Rightarrow v_e' = \sqrt{\frac{GM}{R'}} = \sqrt{4\left(\frac{2GM}{R_e}\right)} = 2\sqrt{\left(\frac{2GM}{R_e}\right)}$$

$$v_e' = 2v_e$$

52. The acceleration of moon with respect to earth is $0{\cdot}0027 ms^{-2}$ and the acceleration of an apple falling on earth's surface is about $10 ms^{-2}$. Assume that the radius of the moon is one fourth of the earth's radius. If the moon is stopped for an instant and then released, it will fall towards the earth. The initial acceleration of the moon towards the earth will be:

a. $10 ms^{-2}$
b. $0.0027 ms^{-2}$
c. $6.4 ms^{-2}$
d. $5.0 ms^{-2}$

Ans. :

b. $0.0027 ms^{-2}$

Explanation:

We know that the distance of the Moon from the Earth is about 60 times the radius of the earth. So, acceleration due to gravity at that distance is $0.0027 m/s^2$. When the Moon is stopped for an instant and then released, it will fall towards the Earth with an initial acceleration of $0.0027 m/s^2$.

53. The time period of an earth-satellite in circular orbit is independent of:

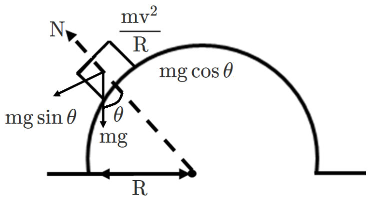
a. The mass of the satellite.
b. Radius of the orbit.
c. None of them.
d. Both of them.

Ans. :

a. The mass of the satellite.

Explanation:

The time period of an earth-satellite in circular orbit is independent of the mass of the satellite, but depends on the radius of the orbit.



54. Inside a uniform spherical shell:
a. The gravitational potential is zero.
b. The gravitational field is zero.
c. The gravitational potential is same everywhere.
d. The gravitational field is same everywhere.

Ans. :

b. The gravitational field is zero.
c. The gravitational potential is same everywhere.
d. The gravitational field is same everywhere.

Explanation:

Inside a uniform spherical shell, the gravitational field is the same everywhere and is equal to zero. The gravitational potential has a constant value inside a uniform spherical shell.

55. Let V and E represent the gravitational potential and field at a distance r from the centre of a uniform solid sphere. Consider the two statements:
a. The plot of V against r is discontinuous.
b. The plot of E against r is discontinuous.
   a. Both A and B are correct.
   b. A is correct but B is wrong.
   c. B is correct but A is wrong.
   d. Both A and B are wrong.

Ans. :

d. Both A and B are wrong.

Explanation:

Both the plots (i.e., V against r and E against r) are continuous curves for a uniform solid sphere.

*** Answer The Following Questions In One Sentence.[1 Marks Each] [8]**

56. Express the constant k of Eq. (7.38) in days and kilometres. Given $k=10^{-13} s^2 m^{-3}$. The moon is at a distance of $3.84 \times 10^5 km$ from the earth. Obtain its time-period of revolution in days.

Ans. : Given

$k=10^{-13} s^2 m^{-3}$

$=10^{-13}\left[\frac{1}{(24 \times 60 \times 60)^2} d^2\right]\left[\frac{1}{(1/1000)^3 km^3}\right]$

$=1.33 \times 10^{-14} d^2 km^{-3}$

Using Eq. (7.38) and the given value of k, the time period of the moon is

$T^2=\left(1.33 \times 10^{-14}\right)\left(3.84 \times 10^5\right)^3$

$T=27.3 d$

57. What would happen to an artificial satellite, if its orbital velocity is slightly decreased due to some defects in it?

Ans. : It will fall onto the Earth.

58. The gravitational force between two spheres is x when the distance between their centers is y. What will be the new force if the separation is made 3y?

Ans. : Since $\mathrm{F} \propto \frac{1}{\mathrm{r}^2}$. Therefore, if r is increased by a factor of 3. F will be reduced by a factor of 9. Thus, the new force will be $\frac{\mathrm{x}}{9}$.

59. What is the orbital velocity for a satellite orbiting close to the surface of earth?

Ans. : $\mathrm{v}_0=\sqrt{\mathrm{gR}}$ for close to earth orbits.

60. Two satellites of masses 3m and m orbit the earth in circular orbits of radii r and 3r respectively. What is the ratio of their orbital speeds?

Ans. : Since, $\mathrm{v}=\sqrt{\frac{\mathrm{GM}}{\mathrm{r}}}$

i.e., $\mathrm{v} \propto \frac{1}{\sqrt{\mathrm{r}}}$

Thus, $\frac{\mathrm{v}_1}{\mathrm{v}_2}=\sqrt{\frac{\mathrm{r}_2}{\mathrm{r}_1}}$

$=\sqrt{\frac{3\mathrm{r}}{\mathrm{r}}}=\sqrt{3}$

61. Two artificial satellites, one close to the surface and other away are revolving around the earth. Which of them has larger speed?

Ans. : Both have same speed of gR.

62. If suddenly the gravitational force of attraction between the earth and a satellite revolving around it becomes zero, then what will happen to the satellite?

Ans. : If suddenly the gravitational force of attraction between the earth and a satellite revolving around it becomes zero, satellite will not be able to revolve around the earth. Instead, the satellite will start moving along a straight line tangentially at that point on its orbit, where it is at the time of gravitational force becoming zero.

63. An artificial satellite is at a height of 36,500km above earth's surface. What is the work done by earth's gravitational force in keeping it in its orbit?

Ans. : Zero.

*** Given Section consists of questions of 2 marks each. [54]**

64. Why do different planets have different escape velocities?

Ans. : Escape velocity, $\mathrm{v}=\sqrt{2\mathrm{gR}}=\sqrt{\frac{2\mathrm{GM}}{\mathrm{R}}}$.

Thus escape velocity of a planet depends upon (i) its mass (M) and (ii) its size (R). As different planets have different masses and sizes, so they have different escape velocities.

65. Answer the following: You can shield a charge from electrical forces by putting it inside a hollow conductor. Can you shield a body from the gravitational influence of nearby matter by putting it inside a hollow sphere or by some other means?

Ans. : No. Electrical forces depend upon the nature of the intervening medium while the gravitational forces don’t depend upon the nature of the intervening medium. So, such shielding acts are not possible in case of gravitation i.e., gravity screens are not possible.

66. Why are space crafts usually launched from west to east? Why is it more advantageous to launch rockets in the equatorial plane?

Ans. : Earth rotates on its axis from west to east. A satellite launched from west to east will have the advantage of the additional velocity of the earth's rotation. The effect is maximum at the equator, hence it is most advantageous to launch the satellite from west to east on the equatorial plane.

67. If earth be at half of its present distance from the sun, how many days will there be in a year?

Ans. : $\mathrm{T}^2\propto \mathrm{r}^3, \text{Since } \mathrm{r}\to\frac{\mathrm{r}}{2}, \frac{\mathrm{T}_1^2}{\mathrm{T}^2}=\left(\frac{1}{2}\right)^3$

$\therefore \mathrm{T}_1=\left(\frac{1}{8}\right)^{\frac{3}{2}}\mathrm{T}; \mathrm{T}_1=\left(\frac{1}{2\sqrt{2}}\right)^3\mathrm{T}$

$\text{i.e.} \mathrm{T}_1=\frac{\mathrm{T}}{16\sqrt{2}}$

68. A black hole is a body from whose surface nothing may ever escape. What is the condition for a uniform spherical mass M to be a black hole? What should be the radius of such a black hole if its mass is the same as that of the Earth?

Ans. : For a body to be a black hole, the escape velocity should be such that even light cannot escape. The limiting case for escape velocity is,

$$\sqrt{\frac{2\mathrm{GM}}{\mathrm{R}}}\leq \mathrm{c}(\text{speed of light})$$

For our earth, M = M_e = 6 × 10^{24}kg,

$$\mathrm{R}=\frac{2\mathrm{GM}}{\mathrm{c}^2}=9\times10^{-2}\mathrm{m} \text{ or } 9\mathrm{cm}$$

69. The moon takes about 27 days to complete one orbit around the earth. The orbit is nearly a circle of radius 3.8 × 10^8m. Calculate the mass of the earth from this data.

Ans. : Let M and m be the mass of the earth and the moon respectively. The gravitational force of attraction provides the centripetal force.

$$\frac{\mathrm{GMm}}{\mathrm{r}^2}=\frac{\mathrm{mv}^2}{\mathrm{r}}$$

$$\Rightarrow \mathrm{M}=\frac{\mathrm{v}^2\mathrm{r}}{\mathrm{G}}=\frac{\omega^2\mathrm{r}^3}{\mathrm{G}}$$

$$=\left(\frac{2\pi}{27\times24\times3600}\right)^2\times\frac{(3.8\times10^8)^3}{6.67\times10^{-11}}$$

$$=5.968\times10^{24}\mathrm{kg}$$

70. What is the height at which the value of g is the same as at a depth of $\frac{\mathrm{R}}{2}$?

Ans. : At depth $\frac{\mathrm{R}}{2}, \mathrm{g}'=\mathrm{g}\left(1-\frac{\mathrm{R}}{2\mathrm{R}}\right)=\frac{\mathrm{g}}{2}$

At height $\mathrm{x}, \mathrm{g}'=\mathrm{g}\left(1-\frac{2\mathrm{x}}{\mathrm{R}}\right)$

$$\therefore \mathrm{g}\left(1-\frac{2\mathrm{x}}{\mathrm{R}}\right)=\frac{\mathrm{g}}{2}$$

$$\frac{1}{2}=\frac{2\mathrm{x}}{\mathrm{R}}$$

$$\therefore \mathrm{x}=\frac{\mathrm{R}}{4}$$

71. A planet reduces its radius by 1% with its mass remaining same. How acceleration due to gravity varies?

Ans. : When mass is same, $\mathrm{g}\propto\frac{1}{\mathrm{R}^2}$.

$$\therefore \frac{\Delta\mathrm{g}}{\mathrm{g}}=2\frac{\Delta\mathrm{R}}{\mathrm{R}}$$

% variation of g is 2%.

72. The value of acceleration due to gravity at the moon is $\frac{1}{6}\mathrm{th}$ of the value of g at the surface of the earth, and the diameter of the moon is $\frac{1}{4}\mathrm{th}$ of the diameter of the earth. Compare the ratio of the escape velocities.

Ans. : $\mathrm{v_e}=\sqrt{\frac{2\mathrm{GM}}{\mathrm{R}}}=\sqrt{2\mathrm{gR}}$

$$=\sqrt{\mathrm{gD}}$$

$$\frac{(\mathrm{v_e})_{\mathrm{moon}}}{(\mathrm{v_e})_{\mathrm{earth}}}=\frac{\sqrt{(\mathrm{gD})_{\mathrm{moon}}}}{\sqrt{(\mathrm{gD})_{\mathrm{earth}}}}$$

$$\sqrt{\frac{1}{6}\times\frac{1}{4}}=\frac{1}{4.9}$$

73. An astronaut, by mistake, drops his food packet from an artificial satellite orbiting around the Earth. Will it reach the surface of Earth? Why?

Ans. : The food packet will not fall on the Earth. As the satellite as well as astronaut were in a state of weightlessness, hence the food packet, when dropped by mistake, will also start moving with the same velocity as that of satellite and will continue to move along with the satellite in the same orbit.

74. Earth's radius is about 6370km. A mass of 20kg is taken to a height of 160km above the earth's surface.

   i. What is the mass of the objects at that height?

ii. How much does the object weigh at this height?

Ans. :

i. The mass of the object remain 20kg at that height.

ii. $W=\frac{GMm}{r^2}$

$\Rightarrow \frac{W_2}{W_1}=\frac{r_1^2}{r_2^2}$ since G, M and m are constant.

$\therefore W_2=9.8\times 20\times\left(\frac{6370}{6370+160}\right)^2=186.5N$

75. The orbiting velocity of an earth-satellite is 8km s. What will be the escape velocity?

Ans. : Escape velocity,

$v_e=\sqrt{2}v_0$

$v_e=\sqrt{2}\times 8=11.31\text{km s}^{-1}$

76. Determine the speed with which the earth has to rotate on its axis so that a person on the equator weigh $\left(\frac{3}{5}\right)^{th}$ as much as at present. Take the equatorial radius as 6400km.

Ans. : Here, $W=mg$

and $W'=\left(\frac{3}{5}\right)mg$

As $W'=W-mR\omega^2$

So, $\frac{3}{5}mg=mg-mR\omega^2$

or $mR\omega^2=mg-\frac{3}{5}mg=\frac{2}{5}mg$

or $\omega=\sqrt{\frac{2g}{5R}}=\sqrt{\frac{2\times 9.8}{5\times 6400\times 10^3}}$

$=4.315\times 10^{-4}\text{rad/s.}$

77. What are the conditions under which a rocket fired from the earth, launches an artificial satellite of earth?

Ans. :

i. Velocity acquired is more than the escape velocity to go out in space.

ii. Provide sufficient velocity to move in a path of its own.

78. What is the acceleration due to gravity at the bottom of a sea 30km deep taking radius of the earth as 6.3 × 10km?

Ans. : $g'=g\left(1-\frac{d}{R}\right)$

$=9.8\left(1-\frac{30\times 1000}{6.3\times(1000)^2}\right)$

$=9.8\left(1-\frac{1}{210}\right)$

$=9.8\left(\frac{209}{210}\right)=9.75ms^{-2}$

79. The escape speed on the earth is 11.2km/ s.What is its value for a planet having double the radius and eight times the mass of the earth?

Ans. : v_p (escap speed on a planet) $= \sqrt{\frac{GM_p}{R_p}}$

v_e (escape speed on the earth) $= \sqrt{\frac{GM_e}{R_e}}$

Clearly, $\frac{v_p}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R\ p}} = \sqrt{8 \times \frac{1}{2}} = 2$

$v_p = 2v_e 22.4 km/s$

80. Determine the speed with which the earth would have to rotate on its axis so that a person on the equator would weigh 3/5th as much as at present. Take the equatorial radius as 6400km.

Ans. : Acceleration due to gravity at the equator is,

$g_e = g - R\omega^2$

$mg_e = mg - mR\omega^2$

$\frac{3}{5}mg = mg - mR\omega^2 \left[\because mg_e = \frac{3}{5}mg\right]$

$\therefore \omega = \sqrt{\frac{2g}{5R}} = \sqrt{\frac{2\times9.8}{6\times6400\times10^3}} = 7.8 \times 10^{-4} rad/s$

81. Imagine what would happen if the value of G becomes:
    i. 100 times of its present value.
    ii. Times of its present value.

Ans. :

i. Earth's attraction would be so large that you would be crushed to the earth.
ii. Earth's attraction would be so less that we can easily jump from the top of a multi-storey building.

82. A 10kg mass is to be divided into two parts, such that the force of attraction between them is maximum. What is the mass of each portion?

Ans. : Let m kg and (10 - m) kg be the mass of two parts separated by a distance r. The force is $F = \frac{Gm(10-m)}{r^2}$

For force to be maximum, $\frac{dF}{dm} = 0$

$\therefore \frac{G}{r^2}[m(-1) + (10 - m \times 1) = 0]$

$\therefore m = 10 - m$

$\therefore m = 5kg$

83. The escape velocity v of a body depends upon:
    i. The acceleration due to gravity 'g' of the planet.
    ii. The radius of the planet 'R'.

Establish dimensionally the relationship between them.

Ans. : $v \propto g^a R^b$

$[LT_{-1}] = K[LT^{-2}]^a[L]^b$

$a = \frac{1}{2}, b = \frac{1}{2}$

$K = \sqrt{2}$

$\therefore v = \sqrt{2gR}$

84. A satellite is revolving just near the Earth's surface. Compute its orbital velocity. Given that radius of Earth R = 6400km and 3 = 9.8ms^{-2}.

Ans. : For a satellite revolving just near the Earth's surface, the orbital velocity has a magnitude given by,

$v_{orb} = \sqrt{gR}$

$\therefore\ v_{orb} = \sqrt{9.8 \times 6400 \times 1000}$

$= 7.92 \times 10^3 ms^{-1}$

$= 7.92 km\ s^{-1}$

85. What is a geostationary satellite? Is it same as geo synchronous satellite?

Ans. : If the revolution period of a satellite is same as that of the earth, it is called a geo-stationary satellite. Yes, geo-synchronous satellites are geo-stationary satellites.

86. A Mars satellite moving in an orbit of radius 9.4 × 10^{3}km takes 27540s to complete one revolution. Calculate the mass of Mars.

Ans. : $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$\Rightarrow 27540 = 2 \times 3.14\sqrt{\frac{(9.4\times10^3\times10^3)^3}{6.67\times10^{-11}\times M}}$

$(27540)^2 = (6.28)^2\frac{(9.4\times10^6)^2}{6.67\times10^{11}\times M}$

$M = \frac{(6.28)^2\times(9.4)^3\times10^{18}}{6.67\times10^{-11}\times(27540)^2} = 6.5 \times 10^{23}kg.$

87. A uniform metal sphere of radius a and mass M is surrounded by a thin uniform spherical shell of equal mass and radius 4a The centre of the shell falls on the surface of the inner sphere. Find the gravitational field at the points P$_1$ and P$_2$ shown in the

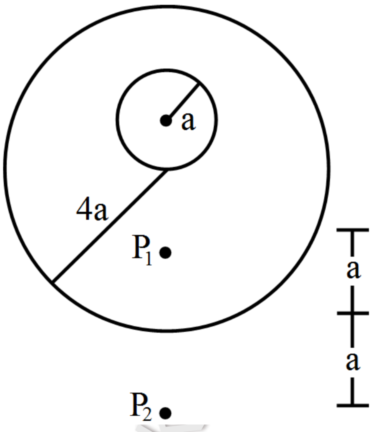


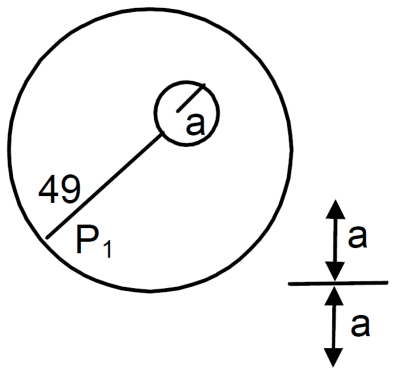
figure.

Ans. : At P$_1$, Gravitational field due to sphere $M = \frac{GM}{(3a+a)^2} = \frac{GM}{16a^2}$

At P$_2$, Gravitational field is due to sphere & shell,

$= \frac{GM}{(a+4a+a)^2} + \frac{GM}{(4a+a)^2}$

$= \frac{GM}{a^2}\left(\frac{1}{36} + \frac{1}{25}\right) = \left(\frac{61}{900}\right)\frac{GM}{a^2}$



88. Two satellites going in equatorial plane have almost same radii. As seen from the earth one moves from east to west and the other from west to east. Will they have the same time period as seen from the earth? If not, which one will have less time period?

Ans. : No, both satellites will have different time periods as seen from the Earth. The satellite moving opposite (east to west) to the rotational direction of the Earth will have less time period, because its relative speed with respect to the Earth is more.

89. Find the acceleration due to gravity in a mine of depth 640m if the value at the surface is 9.800m/s^2. The radius of the earth is 6400km.

Ans. : Let g' be the acceleration due to gravity in mine.

Then, $\mathrm{g'} = \mathrm{g}\left(1 - \frac{\mathrm{d}}{\mathrm{R}}\right)$

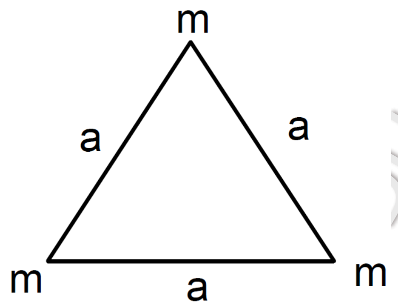
$= 9.8\left(1 - \frac{640}{6400 \times 10^3}\right) - 9.8 \times 0.9999 = 9.799\mathrm{m/s}^2$

90. Three particles of mass m each are placed at the three corners of an equilateral triangle of side a. Find the work which should be done on this system to increase the sides of the triangle to 2a.

Ans. : Initially, the ride of Δ is a

To increase it to 2a,

work done $= \frac{\mathrm{Gm}^2}{2\mathrm{a}} + \frac{\mathrm{Gm}^2}{\mathrm{a}} = \frac{3\mathrm{Gm}^2}{2\mathrm{a}}$



* **Given Section consists of questions of 3 marks each.** **[15]**

91. The time taken by Mars to revolve round the sun is 1.88 years. Find the ratio of average distance between Mars and the sun to that between the earth and the sun.

Ans. : According to Kepler’s laws of planetary motion,

$\mathrm{T}^2 \alpha \mathrm{R}^3$

$\frac{\mathrm{T}_{\mathrm{m}}^2}{\mathrm{T}_{\mathrm{e}}^2} = \frac{\mathrm{R}_{\mathrm{ms}}^3}{\mathrm{R}_{\mathrm{es}}^3}$

$$\left(\frac{R_{ms}}{R_{es}}\right)^3 = \left(\frac{1.88}{1}\right)^2$$

$$\therefore \frac{R_{ms}}{R_{es}} = (1.88)^{\frac{2}{3}} = 1.52$$

92. Two small bodies of masses 2.00kg and 4.00kg are kept at rest at a separation of 2.0m. Where should a particle of mass 0.10kg be placed to experience no net gravitational force from these bodies? The particle is placed at this point. What is the gravitational potential energy of the system of three particles with usual reference level?

Ans. : Let 0.1kg man is x m from 2kg mass and (2 - x) m from 4kg mass.

$\therefore \frac{2\times0.1}{x^2} = -\frac{4\times0.1}{(2-x)^2}$

or $\frac{0.2}{x^2} = \frac{2}{(2-x)^2}$ or $(2-x)^2 = 2x^2$

or $\frac{1}{x^2} = \frac{2}{2x^2}$ or $(2-x)^2 = 2x^2$

or $2 - x = \sqrt{2}$ or $x(r_2 + 1) = 2$

or $x = \frac{2}{2.414} = 0.83m$ from 2kg mass.

93. Two small bodies of masses 10kg and 20kg are kept a distance 1.0m apart and released, Assuming that only mutual gravitational forces are acting, find the speeds of the particles when the separation decreases to 0.5m.

Ans. : The linear momentum of 2 bodies is 0 initially. Since gravitational force is internal, final momentum is also zero.

So (10kg)v_1 = (20kg)v_2

Or v_1 = v_2 ...(1)

Since P.E. is conserved

Initial P.E. $= \frac{-6.67\times10^{-11}\times10\times20}{1} = -13.34\times10^{-9}J$

When separation is 0.5m,

$$-13.34\times10^{-9} + 0 = \frac{-13.34\times10^{-9}}{\left(\frac{1}{2}\right)}$$

$$+\left(\frac{1}{2}\right)\times10v_1^2 + \left(\frac{1}{2}\right)\times20v_2^2\ldots(2)$$

$$\Rightarrow -13.34\times10^{-9} = -26.68\times10^{-9} + 5v_1^2 + 10v_2^2$$

$$\Rightarrow -13.34\times10^{-9} = -26.68\times10^{-9} + 30v_2^2$$

$$\Rightarrow v_2^2 = \frac{13.34\times10^{-9}}{30} = 4.44\times10^{-10}$$

$$\Rightarrow v_2 = 2.1\times10^{-5}m/s.$$

So, $v_1 = 4.2\times10^{-5}m/s.$

94. The moon takes about 27.3 days to revolve round the earth in a nearly circular orbit of radius 3.84 x 10^5km. Calculate the mass of the earth from these data.

Ans. : $T = 2\pi\sqrt{\frac{r^3}{GM}}$

$$27.3 = 2\times3.14\sqrt{\frac{(3.84\times10^5)^3}{6.67\times10^{-11}\times M}}$$

or $2.73 \times 2.73 = \frac{2 \times 3.14 \times (3.84 \times 10^5)^3}{6.67 \times 10^{-11} \times M}$

or $M = \frac{2 \times (3.14)^2 \times (3.84)^3 \times 10^{15}}{3.335 \times 10^{11} (27.3)^2} = 6.02 \times 10^{24} kg$

$\therefore$ mass of earth is found to be 6.02×10^{24}kg.

95. What is the true weight of an object in a geostationary satellite that weighed exactly 10.0N at the north pole?

Ans. : For geo stationary satellite,

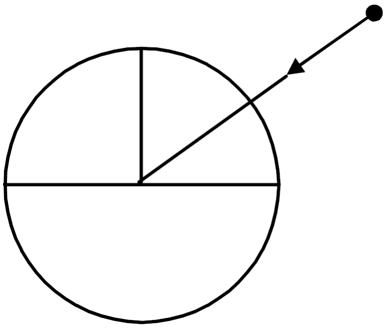
$r = 4.2 \times 10^4$km

$h = 3.6 \times 10^4$km

Given mg = 10N

$$mgh = Mg\left(\frac{R^2}{(R+h)^2}\right)$$

$$= 10\left[\frac{\left(6400 \times 10^3\right)^2}{\left(6400 \times 10^3 + 3600 \times 10^3\right)^2}\right] = \frac{4096}{17980} = 0.23N$$



* **Given Section consists of questions of 5 marks each.** **[150]**

96. Two uniform solid spheres of equal radii R, but mass M and $4M$ have a centre to centre separation $6R$, as shown in Fig. 7.10. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the centre of the second sphere. Obtain an expression for the minimum speed v of the projectile so that it reaches the surface of the second sphere.

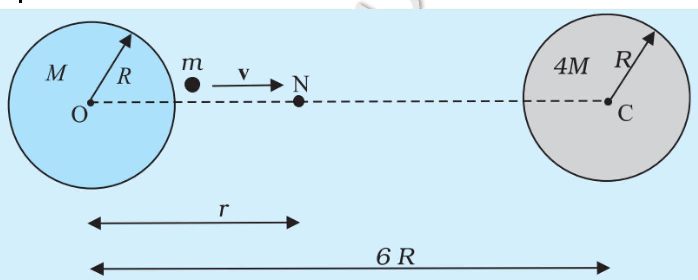


Fig. 7.10

Ans. : The projectile is acted upon by two mutually opposing gravitational forces of the two spheres. The neutral point N (see Fig. 7.10) is defined as the position where the two forces cancel each other exactly. If $ON = r$, we have

$\frac{GMm}{r^2} = \frac{4GMm}{(6R-r)^2}$

$(6R-r)^2 = 4r^2$

$6R - r = \pm 2r$

$r = 2R \text{ or } -6R.$

The neutral point $r = -6R$ does not concern us in this example. Thus $ON = r = 2R$. It is sufficient to project the particle with a speed which would enable it to reach N. Thereafter, the greater gravitational pull of $4M$ would suffice. The mechanical energy at the surface of M is

$E_i = \frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{4GMm}{5R}.$

At the neutral point N, the speed approaches zero. The mechanical energy at N is purely potential.

$E_N = -\frac{GMm}{2R} - \frac{4GMm}{4R}.$

From the principle of conservation of mechanical energy

$\frac{1}{2}v^2 - \frac{GM}{R} - \frac{4GM}{5R} = -\frac{GM}{2R} - \frac{GM}{R}$

or

$v^2 = \frac{2GM}{R}\left(\frac{4}{5} - \frac{1}{2}\right)$

$v = \left(\frac{3GM}{5R}\right)^{1/2}$

97. A star 2.5 times the mass of the sun and collapsed to a size of 12km rotates with a speed of 1.2 rev. per second. (Extremely compact stars of this kind are known as neutron stars. Certain stellar objects called pulsars belong to this category). Will an object placed on its equator remain stuck to its surface due to gravity? (mass of the sun = 2×10^{30}kg).

Ans. : A body gets stuck to the surface of a star if the inward gravitational force is greater than the outward centrifugal force caused by the rotation of the star.

Gravitational force, $f_g = -GMm/R^2$

Where,

M = Mass of the star = $2.5 \times 2 \times 10^{30} = 5 \times 10^{30}$kg

m = Mass of the body

R = Radius of the star = 12km = 1.2×10^4m

$\therefore f_g = 6.67 \times 10^{-11} \times 5 \times 10^{30} \times m/(1.2 \times 10^4)^2 = 2.31 \times 10^{11}$m N

Centrifugal force, $f_c = mr\omega^2$

$\omega =$ Angular speed $= 2\pi v$

ν = Angular frequency = 1.2rev s^{-1}

$f_c = mR(2\pi v)^2$

$= m \times (1.2 \times 10^4) \times 4 \times (3.14)^2 \times (1.2)^2 = 1.7 \times 10^5$m N

Since $f_g > f_c$, the body will remain stuck to the surface of the star.

98. A rocket is fired 'vertically' from the surface of mars with a speed of 2km s^{-1}. If 20% of its initial energy is lost due to martian atmospheric resistance, how far will the rocket go from the surface of mars before returning to it? Mass of mars = 6.4×10^{23}kg; radius of mars = 3395km; G = 6.67×10^{-11}N m^2 kg^{-2}.

Ans. : Initial velocity of the rocket, v = 2km/s = 2×10^3m/s

Mass of Mars, M = 6.4×10^{23}kg

Radius of Mars, R = 3395km = 3.395×10^6m

Universal gravitational constant, G = 6.67×10^{-11}N m^2 kg^{-2}

Mass of the rocket = m

Initial kinetic energy of the rocket = $(1/2)mv^2$

Initial potential energy of the rocket = -GMm/R

Total initial energy = $(1/2)mv^2$ - GMm/R

If 20% of initial kinetic energy is lost due to Martian atmospheric resistance, then only 80% of its kinetic energy helps in reaching a height.

Total initial energy available = $(80/100) \times (1/2)\ mv^2$ - GMm/R = $0.4mv^2$ - GMm/R

Maximum height reached by the rocket = h

At this height, the velocity and hence, the kinetic energy of the rocket will become zero.

Total energy of the rocket at height h = -GMm/(R + h)

Applying the law of conservation of energy for the rocket, we can write:

$0.4mv^2 - GMm/R = -GMm/(R + h)$

$0.4v^2 = GM/R - GM/(R + h)$

$= GMh/R(R + h)$

$(R + h)/h = GM/0.4v^2R$

$R/h = (GM/0.4v^2R) - 1$

$h = R/[(GM/0.4v^2R) - 1]$

$= 0.4R^2v^2 / (GM - 0.4v^2R)$

=

$0.4 \times (3.395 \times 10^6)^2 \times (2 \times 10^3)^2/[6.67 \times 10^{11} \times 6.4 \times 10^{23} - 0.4 \times (2 \times 10^3)^2 \times (3.395 \times 10^6)]$

$= 18.442 \times 10^{18}/[42.688 \times 10^{12} - 5.432 \times 10^{12}]$

$= 18.442 \times 10^6/37.256$

$= 495 \times 10^3$m = 495km.

99. The escape speed of a projectile on the earth's surface is 11.2km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Ans. : Escape velocity of a projectile from the Earth, v_{esc} = 11.2km/s

Projection velocity of the projectile, $v_p = 3v_{esc}$

Mass of the projectile = m

Velocity of the projectile far away from the Earth = v_f

Total energy of the projectile on the Earth = $(1/2)mv_p^2 - (1/2)mv_{esc}^2$

Gravitational potential energy of the projectile far away from the Earth is zero.

Total energy of the projectile far away from the Earth = $(1/2)mv^2{}_f$

From the law of conservation of energy, we have

$$\frac{1}{2}mv_p^2 - \frac{1}{2}mv_{esc}^2 = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{v_p^2 - v_{esc}^2}$$

$$= \sqrt{(3v_{esc})^2 - (v_{esc})^2}$$

$$= \sqrt{8}v_{esc}$$

$$= \sqrt{8} \times 11.2 = 31.68 km/s$$

100. A body weighs 63N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

Ans. : Weight of the body, W = 63N

Acceleration due to gravity at height h from the Earth's surface is given by the relation:

$$g' = \frac{g}{\left[1+\left(\frac{h}{R_e}\right)\right]^2}$$

Where,

g = Acceleration due to gravity on the Earth's surface

R_e = Radius of the Earth

For $h = \frac{R_e}{2}$

$$g' = \frac{g}{\left[1+\left(\frac{R_e}{2R_e}\right)\right]^2}$$

$$= \frac{g}{\left[1+\left(\frac{1}{2}\right)\right]^2} = \left(\frac{4}{9}\right)g$$

Weight of a body of mass m at height h is given as:

W' = mg

$$= m \times \left(\frac{4}{9}\right)g = \left(\frac{4}{9}\right)mg$$

$$= \left(\frac{4}{9}\right)W$$

$$= \left(\frac{4}{9}\right) \times 63 = 28N.$$

101. A satellite orbits the earth at a height of 400km above the surface. How much energy must be expended to rocket the satellite out of the earth's gravitational influence? Mass of the satellite = 200kg; mass of the earth = 6.0×10^{24}kg; radius of the earth = 6.4×10^6m; G = 6.67×10^{-11}N m^2 kg^{-2}.

Ans. : Mass of the Earth, M = 6.0×10^{24}kg

Mass of the satellite, m = 200kg

Radius of the Earth, $R_e = 6.4 \times 10^6$m

Universal gravitational constant, G = 6.67×10^{-11}Nm^2 kg^{-2}

Height of the satellite, h = 400km = 4×10^5m = 0.4×10^6m

Total energy of the satellite at height $h = \left(\frac{1}{2}\right)mv^2 + \left[\frac{-GM_em}{(R_e+h)}\right]$

Orbital velocity of the satellite, $\mathrm{v} = \left[\frac{\mathrm{GM_e}}{(\mathrm{R_e+h})}\right]^{\frac{1}{2}}$

Total energy of height, $\mathrm{h} = \frac{\left(\frac{1}{2}\right)\mathrm{GM_e m}}{(\mathrm{R_e+h})} - \frac{\mathrm{GM_e m}}{(\mathrm{R_e+h})}$

$= -\frac{\left(\frac{1}{2}\right)\mathrm{GM_e m}}{(\mathrm{R_e+h})}$

The negative sign indicates that the satellite is bound to the Earth. This is called bound energy of the satellite.

Energy required to send the satellite out of its orbit = -(Bound energy)

$= \frac{\left(\frac{1}{2}\right)\mathrm{GM_e m}}{(\mathrm{R_e+h})}$

$= \frac{\left(\frac{1}{2}\right)\times 6.67\times 10^{-11}\times 6\times 10^{24}\times 200}{(6.4\times 10^{6}+0.4\times 10^{6})}$

$= 5.9 \times 10^{9}$ j.

102. A saturn year is 29.5 times the earth year. How far is the saturn from the sun if the earth is 1.50 × 10^8km away from the sun?

Ans. : Distance of the Earth from the Sun, re = 1.5 × 10^8km = 1.5 × 10^{11}m

Time period of the Earth = T_e

Time period of Saturn, T_s = 29.5T_e

Distance of Saturn from the Sun = r_s

From Kepler's third law of planetary motion, we have

$\mathrm{T} = \left(\frac{4\pi^2 \mathrm{r}^3}{\mathrm{GM}}\right)^{\frac{1}{2}}$

For Saturn and Sun, we can write

$\frac{\mathrm{r_s^3}}{\mathrm{r_e^3}} = \frac{\mathrm{T_s^2}}{\mathrm{T_e^2}}$

$\mathrm{r_s} = \mathrm{r_e}\left(\frac{\mathrm{T_s}}{\mathrm{T_e}}\right)^{\frac{2}{3}}$

$= 1.5 \times 10^{11}\left(\frac{29.5\mathrm{T_e}}{\mathrm{T_e}}\right)^{\frac{2}{3}}$

$= 1.5 \times 10^{11}(29.5)^{\frac{2}{3}}$

$= 1.5 \times 10^{11} \times 9.55$

$= 14.32 \times 10^{11}\mathrm{m}$

Hence, the distance between Saturn and the Sun is 1.43 × 10^{12}m.

103. Earth's orbit is an ellipse with eccentricity 0.0167. Thus, earth's distance from the sun and speed as it moves around the sun varies from day to day. This means that the length of the solar day is not constant through the year. Assume that earth's spin axis is normal to its orbital plane and find out the length of the shortest and the longest day. A day should be taken from noon to noon. Does this explain variation of length of the day during the year?

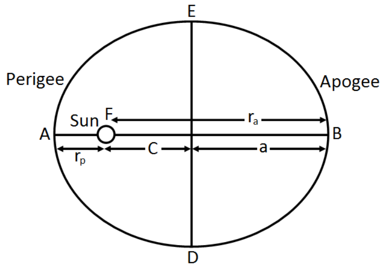
Ans. :

According to the diagram,

r_p = radius of perigee = 2R

r_a = radius of apogee = 6R

a = semi - major axis of the ellipse



Hence, we can write

r_a = a(1 + e) = 6R

P_p = a(1 - e) = 2R

$\frac{a(1+e)}{a(1-e)} = \frac{6R}{2R} = 3$

By solving, we get eccentricity $e = \frac{1}{2}$

If v_a and v_p are the velocities of the satellite (of mass m) at aphelion and perihelion respectively, then by conservation of angular momentum

$\therefore\ L_{\text{at perigee}} = L_{\text{at apogee}}$

$mv_p r_p = mv_a r_a$

$\therefore\ \frac{v_a}{v_p} = \frac{r_p}{r_a} = \frac{1}{3}$

Applying conservation of energy,

Energy at perigee = Energy at apogee

where M is the mass of the earth

$\therefore\ v_p^2\left(1 - \frac{1}{9}\right) = -2GM\left(\frac{1}{r_a} - \frac{1}{r_p}\right)$

$= 2GM\left(\frac{1}{r_p} - \frac{1}{r_a}\right)$ (By putting $v_a = \frac{v_p}{3}$)

$$v_p = \frac{\left[2GM\left(\frac{1}{r_p} - \frac{1}{r_a}\right)\right]^{\frac{1}{2}}}{\left[1 - \left(\frac{v_a}{v_p}\right)^2\right]^{\frac{1}{2}}}$$

$$= \left[\frac{\frac{2GM}{R}\left(\frac{1}{2} - \frac{1}{6}\right)}{\left(1 - \frac{1}{9}\right)}\right]^{\frac{1}{2}} = \left(\frac{\frac{2}{3}}{\frac{8}{9}}\frac{GM}{R}\right)^{\frac{1}{2}} = \sqrt{\frac{3}{4}\frac{GM}{R}} = 6.85\text{km/ s}$$

$v_p = 6.85\text{km/ s}$, $v_a = 2.28\text{km/ s}$,

For circular orbit of radius r,

v_c = orbital velocity $= \sqrt{\frac{GM}{r}}$

For r = 6R, $v_c = \sqrt{\frac{GM}{6R}}$ = 3.23km/ s

104. A satellite is in an elliptic orbit around the earth with aphelion of 6R and perihelion of 2R where R = 6400km is the radius of the earth. Find eccentricity of the orbit. Find the

velocity of the satellite at apogee and perigee. What should be done if this satellite has to be transferred to a circular orbit of radius 6R? [G = 6.67 × 10^{-11} SI units and M = 6 × 10^{24}kg]

Ans. : r_p = 2R r_a =6R

Hence, r_p = a(1 - e) = 2R ...(i)

r_a = a(1 + e) = 6R ...(ii)

on dividing (i) by (ii)

$\frac{1-e}{1+e} = \frac{2}{6}$

$3 - 3e = 1 + e$

$4e = 2 \Rightarrow e = \frac{1}{2}$

There is not external force or torque on system.

So by the law of conservation of angular momentum.

$L_1 = L_2$

$m_a v_a r_a = m_p v_p r_p$ $m_a = m_p = m =$ mass of satellite

$\therefore \frac{v_a}{v_p} = \frac{r_p}{r_a} \frac{2R}{6R} = \frac{1}{3}$

So, $v_p = 3v_a$

Apply conservation of energy at apogee an d perigee ...(iii)

$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a}$

Multiplying $\frac{2}{m}$ to both side and putting r_p = 2R and r_a = 6R

$v_p^2 - \frac{2GM}{2R} = v_a^2 - \frac{2GM}{6R}$ (where M is mass of earth)

$v_a = \frac{v_p}{3}$

$v_p^2 - v_a^2 = \frac{GM}{R} - \frac{1}{3}\frac{GM}{R}$

$v_p^2 - \left(\frac{v_p}{3}\right)^2 = \frac{GM}{R}\left[1 + \frac{1}{3}\right]$

$v_p^2 \frac{8}{9} = \frac{GM}{R} . \frac{2}{3}$

$v_p^2 = \frac{GM}{R}\frac{2}{3} \times \frac{9}{8} = \frac{3}{4}\frac{GM}{R}$

$v_p = \sqrt{\frac{3}{4}\frac{GM}{R}} = \sqrt{\frac{3 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{4 \times 6.6 \times 10^{6}}}$

$= \sqrt{\frac{9 \times 667 \times 10^{24-6-11-1}}{128}}$

$v_p = \sqrt{\frac{6003 \times 10^{18-11-1}}{128}} = \sqrt{46.89 \times 10^6}$

$= 6.85 \times 10^3 m/s = 6.85 km/s$

$v_a = \frac{v_p}{3} = \frac{6.85}{3} = 2.28 km/s$

$v_c = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6R}}$

$= \sqrt{\frac{6.67 \times 6 \times 10^{24-11}}{6 \times 6.8 \times 10^{6}}} = \sqrt{\frac{667}{640} \times 10^{13-6}}$

$= \sqrt{1.042 \times 10 \times 10^6} = \sqrt{10.42 \times 10^6}$

$v_c = 3.23 km/s$

Hence to transfer to a circular orbit at apogee we have to boost the velocity by

$$\mathrm{v_0 - v_a = (3.23 - 2.28) = 0.95 km/\ s}$$

105. A spaceship is stationed on Mars. How much energy must be expended on the spaceship to launch it out of the solar system? Mass of the space ship = 1000kg; mass of the sun = 2×10^{30}kg; mass of mars = 6.4×10^{23}kg; radius of mars = 3395km; radius of the orbit of mars = 2.28×10^{8}km; G = 6.67×10^{-11}N m^2 kg^{-2}.

Ans. : Mass of the spaceship, m_s = 1000kg

Mass of the Sun, M = 2×10^{30}kg

Mass of Mars, mm = 6.4×10^{23}kg

Orbital radius of Mars, R = 2.28×10^{8}kg =2.28×10^{11}m

Radius of Mars, r = 3395km = 3.395×10^{6}m

Universal gravitational constant, G = $6.67 \times 10^{-11} m^2\ kg^{-2}$

Potential energy of the spaceship due to the gravitational attraction of the Sun = $-GMm_s/R$

Potential energy of the spaceship due to the gravitational attraction of Mars = $-GM_m m_s/r$

Since the spaceship is stationed on Mars, its velocity and hence, its kinetic energy will be zero.

Total energy of the spaceship = $(-GMm_s)/R - (-GM_m m_s)/r = -Gm_s(M/R + m_m/r)$

The negative sign indicates that the system is in bound state.

Energy required for launching the spaceship out of the solar system

= -(Total energy of the spaceship)

$$= \mathrm{Gm_s}\left(\frac{\mathrm{M}}{\mathrm{R}} + \frac{\mathrm{m_m}}{\mathrm{r}}\right)$$

$$= 6.67 \times 10^{-11} \times 10^{3} \times \left(\frac{2\times10^{30}}{2.28\times10^{11}} + \frac{6.4\times10^{23}}{3.395\times10^{6}}\right)$$

$$= 6.67 \times 10^{-8}(87.72 \times 10^{17} + 1.88 \times 10^{17})$$

$$= 6.67 \times 10^{-8} \times 89.50 \times 10^{17}$$

$$= 596.97 \times 10^{9}$$

$$= 6 \times 10^{11}\mathrm{J}$$

106. Define gravitational potential energy of a body. Derive an expression for the gravitational potential energy of a body of mass 'm' located at a distance 'r' from the centre of the earth.

Ans. : Gravitational potential energy. The work done in carrying a mass 'm' from infinity to a point at distance r is called gravitational potential energy.

$$\mathrm{G.P.E.} = -\frac{\mathrm{GMm}}{\mathrm{r}}$$

i.e., G.P.E. = Mass × Gravitational potential

It is a scalar quantity measured in joule. Negative sign means that the mass is bound to M.

The gravitational force of attraction between M and m when x is the distance between their centres is given by,

$$\mathrm{F} = \frac{\mathrm{GMm}}{\mathrm{x^2}}$$

Suppose the body is moved through a distance dx, therefore, work done is given by,

$$dW = Fdx = \frac{GMm}{x^2}dx$$

When the body is brought from infinity to some distance r,

We write, $\int dW = \int_{x=\infty}^{x=r} \frac{GMm}{x^2}dx$

$$\text{or } W = GMm\left[\frac{-1}{x}\right]_{\infty}^{r}$$

$$= -GMm\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{-GMm}{r}$$

This amount of work done is the change in the potential energy of the body.

$$\therefore \text{P.E. U } = \frac{-GMm}{r}$$

107. Obtain an expression for the escape velocity of an object of mass m from the surface of a planet of mass M and radius R. For planet earth, escape velocity is known to have a value of 11.2km/s. How fast will an object be moving when at infinity if it is launched with a speed of 22.4km/s from the surface of the earth?

Ans. : The minimum velocity required to escape from the gravitational force of earth is called escape velocity. Total energy is the sum of P.E. and K.E.

$$\text{T.E} = \frac{-GMm}{R} + \frac{1}{2}mv^2$$

To escape, K.E. should be greater than P.E., i.e.,

$$\frac{1}{2}mv^{2t} \geq \frac{GMm}{R}$$

$$v_e = \sqrt{2\frac{GM}{R}} = \sqrt{2gR}$$

Intial velocity of the object(m)

$$= 22.4\text{km/s} = 2v_e$$

P.E. on the surface of earth

Conserving energy we get

$$= -\frac{GM_em}{R_e} + \frac{1}{2}m(2v_e)^2$$

$$= -\frac{GM_em}{x} + \frac{1}{2}mv^2$$

$$\therefore -\frac{GM_em}{R_e} + \frac{1}{2}m4\frac{GM_e}{R_e} = \frac{1}{2}mv^2$$

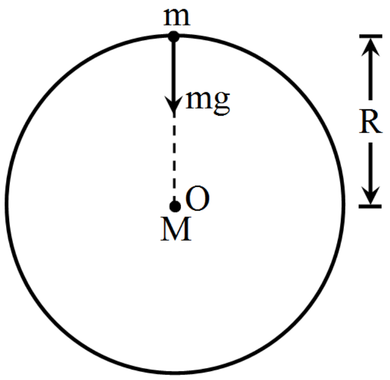
$$\left(\because -\frac{GM_em}{x} = 0 \text{ as far point is at infinity.}\right)$$

$$-\frac{2GM_e}{R_e} = \sqrt{2gR_e} = v_e = 11.2\text{km/sec}$$

108. Obtain an expression for the acceleration due to gravity on the surface of the earth in terms of man of the earth and its radius. Discuss the variation of acceleration due to gravity with altitude and depth. If a body is taken to a height equal to $\frac{R}{4}$ from the surface of the earth then find percentage decrease in the weight of the body? Where R is radius of the earth.

Ans. : Consider the mass of earth M, radius R with centre 0. Suppose a body of mass m placed on the surface of earth, where acceleration due to gravity is g.

The force on body of mass m, outside the surface of earth is due to earth whose mass M is concentrated at the centre O.



Let F be the force of attraction between body and the earth. According to Newton's law of gravitation.

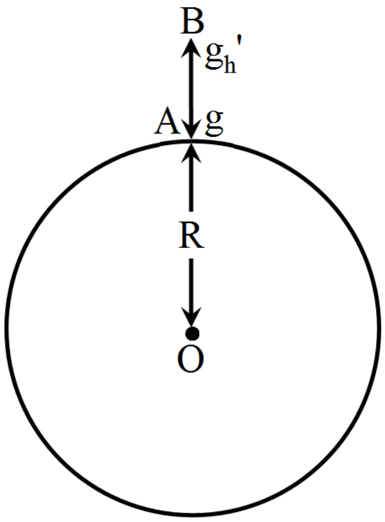
$F = \frac{GMm}{x^2}$

Form gravity pull, F = mg

$\therefore mg = \frac{GMm}{R^2}$

$\Rightarrow g = \frac{GM}{R^2}$

Effect of altitude:



$g_h' = g\left(1 - \frac{2h}{R}\right) \text{when } h << R$

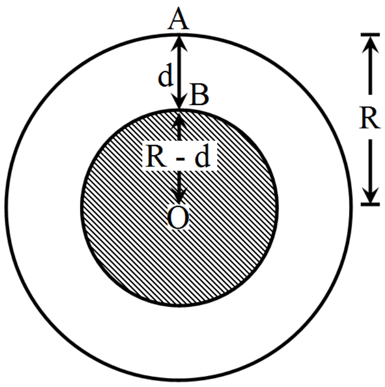
So, the value of acceleration due to gravity (g) decrease with height.

Effect of depth:

$g_d' = g\left(1 - \frac{d}{R}\right)$

So, the value of acceleration due to gravity (g) decreases with depth.

As body is taken to a height equal to $\frac{R}{4}$ from the surface of the earth.



$$\therefore \frac{g_h}{g} = \frac{R^2}{\left(R+\frac{R}{4}\right)^2}$$

$$= \frac{R^2}{\left(\frac{5SR}{4}\right)^2} = \frac{16}{25}$$

weight of body at altitude h is = mg'$_h$

% decrease in weight $= \frac{mg-mg'}{mg} \times 100$

$$= 1 - \frac{g'}{g} \times 100$$

$$= \left(1 - \frac{16}{25}\right) \times 100 = \frac{9}{25} \times 100$$

$$= 36\%$$

109. A rocket is launched vertically from the surface of the earth with an initial speed of 10km/s. How far above the surface of the earth would it go? Ignore the atmospheric resistance. Radius of the earth = 6400km.

Ans. : Total energy on the surface of the earth

$$= \text{K.E.} + \text{P.E.}$$

$$= \frac{1}{2}mv^2 - \frac{GmM}{R}$$

KE = 0 at the highest point, therefore,

$$\text{P.E.} = \frac{GmM}{R+h},$$

where h is the maximum height attained. Using the law of conservation of energy,

$$\frac{1}{2}mv^2 - \frac{GmM}{R} = -\frac{GmM}{R+h}$$

$$= GmM\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$\text{or } v^2 = \frac{2gRh}{(R+h)}$$

$$\text{or } h = R\left(\frac{2gR}{v^2} - 1\right)^{-1}$$

$$= R\left(\frac{2\times9.8\times6.4\times10^6}{10^8} - 1\right)^{-1}$$

$$\text{or } h = 3.93R = 2.5 \times 10^7 \text{m.}$$

110. Jupiter has a mass 318 times that of the earth, and its radius is 11.2 times the earth's radius. Estimate the escape velocity of a body from Jupiter's surface given that the escape velocity from the earth's surface is 11.2km s^{-1}.

Ans. : Escape Velocity from the earth's surface is,

$$\sqrt{\frac{2GM}{R}} = 11.2\ (\text{given}) \ldots (1)$$

Now, escape velocity from Jupiter's surface will be

$$v_e = \sqrt{\frac{2Gm'}{R}}$$

$$\text{But } m' = 318m \text{ and } R' = 11.2R$$

$$\text{Hence, } v_e = \sqrt{\frac{2G(318m)}{11.2R}}$$

$$= \sqrt{\frac{2Gm}{R}} \times \sqrt{\frac{318}{11.2}}$$

$$= 11.12 \times \sqrt{\frac{318}{11.2}} = 59.7 kms^{-1}$$

111. An artificial satellite of mass 1000kg revolves around the earth in circular orbit of radius 6500km. Calculate,

i. Orbital velocity.
ii. Orbital kinetic energy.
iii. Gravitational potential energy.
iv. Total energy in the orbit. [Mass of the earth = 6 × 1024kg, G = 6.67 × 10-11 Nm^2/ kg]

Ans. :

i. Orbital velocity $v_0 = \sqrt{\frac{GM}{r}}$, where M is the mass of earth and r the radius of the path.

$$\therefore v_o = \sqrt{\frac{6.67 \times 10 - 11 \times 6 \times 10^{24}}{6500 \times 10^3}}$$

$$= 6.16 \times 10^7 ms^{-1}$$

ii. Orbital K.E. $= \frac{1}{2} m v_o^2 = \frac{1}{2} m \frac{GM}{r}$

$$= \frac{1}{2} \times 10^3 \times \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6500 \times 10^3}$$

$$\text{K.E.} = \frac{61.67 \times 10^{11}}{2} = 30.84 \times 10^{11} J$$

iii. Gravitational potential energy

$$= \frac{-GMm}{r} = -2 \times \text{K.E.}$$

iv. Total energy in orbit = P.E. + K.E.

$$= -\text{K.E.} = -30.84 \times 10^{11} J$$

112. The escape speed of a projectile on the earth’s surface is 11.2km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Ans. : Escape velocity of a projectile from the Earth, v_{esc} = 11.2km/s

Projection velocity of the projectile, $v_p = 3v_{esc}$

Mass of the projectile = m

Velocity of the projectile far away from the Earth = v_f

Total energy of the projectile on the Earth = $(1/2)mv_p^2 - (1/2)mv_{esc}^2$

Gravitational potential energy of the projectile far away from the Earth is zero.

Total energy of the projectile far away from the Earth = $(1/2)mv^2_f$

From the law of conservation of energy, we have

$$\frac{1}{2}\mathrm{mv}_\mathrm{p}^2-\frac{1}{2}\mathrm{mv}_\mathrm{esc}^2=\frac{1}{2}\mathrm{mv}_\mathrm{f}^2$$

$$\mathrm{v_f}=\sqrt{\mathrm{v}_\mathrm{p}^2-\mathrm{v}_\mathrm{esc}^2}$$

$$=\sqrt{(3\mathrm{v}_\mathrm{esc})^2-(\mathrm{v}_\mathrm{esc})^2}$$

$$=\sqrt{8}\mathrm{v}_\mathrm{esc}$$

$$=\sqrt{8}\times 11.2=31.68\mathrm{km/s}$$

113. A body weighs 63N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

Ans. : Weight of the body, W = 63N

Acceleration due to gravity at height h from the Earth's surface is given by the relation:

$$\mathrm{g}'=\frac{\mathrm{g}}{\left[1+\left(\frac{\mathrm{h}}{\mathrm{R_e}}\right)\right]^2}$$

Where,

g = Acceleration due to gravity on the Earth's surface

R_e = Radius of the Earth

For $\mathrm{h}=\frac{\mathrm{R_e}}{2}$

$$\mathrm{g}'=\frac{\mathrm{g}}{\left[1+\left(\frac{\mathrm{R_e}}{2\mathrm{R_e}}\right)\right]^2}$$

$$=\frac{\mathrm{g}}{\left[1+\left(\frac{1}{2}\right)\right]^2}=\left(\frac{4}{9}\right)\mathrm{g}$$

Weight of a body of mass m at height h is given as:

W' = mg

$$=\mathrm{m}\times\left(\frac{4}{9}\right)\mathrm{g}=\left(\frac{4}{9}\right)\mathrm{mg}$$

$$=\left(\frac{4}{9}\right)\mathrm{W}$$

$$=\left(\frac{4}{9}\right)\times 63=28\mathrm{N}.$$

114. What happens to a body when it is projected vertically upwards from the surface of the earth with a speed of 11200m/s, and why? Compare escape speeds for two planets of masses M and 4M and radii 2R and R respectively.

Ans. : Speed of projection = 11200m/s = 11.2km/s

Since this is equal to the escape velocity, the mass thrown should escape from the surface of earth. Escape speed on the surface of a planet is given by,

$$\mathrm{v_e}=\sqrt{2\mathrm{gR}}$$

$$\Rightarrow \mathrm{v_e}=\sqrt{\frac{2\mathrm{GM}}{\mathrm{R}}}$$

where M and R are the mass and radius of the planet.

$$\mathrm{M_1}=\mathrm{M_2}=1:4$$

$$R_1 : R_2 = 2 : 1$$

$$\therefore \frac{v_{e1}}{v_{e2}} = \sqrt{\frac{M_1}{M_2}\frac{R_2}{R_1}} = \sqrt{\frac{1}{4}} \times \frac{1}{2}$$

$$= \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}}$$

115. A mass m is left free at a distance 3R from the surface of earth. On reaching the surface, what will be its velocity?

Ans. : Initial energy $= -\frac{GM_m}{4R}$

On reaching the surface of earth.

$$\text{Energy} = \frac{GM_m}{R} + \frac{1}{2}\ mv^2$$

$$\therefore -\frac{GM_m}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{4R}$$

$$v^2 = 2\left[-\frac{GM_m}{R}\left(\frac{1}{4} - 1\right)\right]$$

$$\therefore\ v = \sqrt{\frac{3\ GM_m}{2\ R}}\ ms^{-1}$$

116. A geostationary satellite is orbiting the earth at a height of 6R above the surface of earth. Here R is the radius of the earth. What is the time period of another satellite at a height of 2.5R from the surface of the earth?

Ans. : In first case, radius or orbit R + 6R = 7R

In second case, redius of orbit R + 2.5R = 3.5R

$$\frac{T'^2}{T'^2} = \frac{R'^3}{R^3}$$

$$\Rightarrow\ T'^2 = \left(\frac{R'}{R}\right)^3 T^2$$

$$\Rightarrow\ T'^2 = \left[\frac{3.5R}{7R}\right]^3 \times 24 \times 24$$

$$\Rightarrow\ T' = \frac{24}{\sqrt{8}}hs = \frac{12}{1.414}hr$$

$$\Rightarrow T' = 8.49hr.$$

117. If the earth supposed to be a uniform sphere, contracts slightly so that its radius becomes less by $\left(\frac{R}{n}\right)$ than before, show that the length of the day shortens by $\left(\frac{48}{n}\right)$ hours.

Ans. : The present length of day, T = 24 hours;

$$\omega = \frac{2\pi}{T}$$

Let M be the mass of the earth, R and R' be the radii of earth before and after contraction. Then

$$R' = R - \frac{R}{n} = R\left(1 - \frac{1}{n}\right)$$

According to law of conservation of angular momentum,

$$I\omega = I'\omega'$$

$$\text{or } \frac{2}{5}\ MR^2\omega = \frac{2}{5}MR'^2\omega'^2$$

$$\text{or } \omega' = \frac{R^2\omega}{R'^2}$$

$$= \frac{R^2\omega}{R^2\left[1-\left(\frac{1}{n}\right)\right]^2}$$

$$= \left(1-\frac{1}{n}\right)^{-2}\omega = \left(1+\frac{2}{n}\right)\omega$$

$$\therefore T' = \frac{2\pi}{\omega'} = \frac{2\pi}{\left[1+\left(-\frac{2}{n}\right)\right]\omega}$$

$$= \frac{2\pi}{\omega}\left(1+\frac{2}{n}\right)^{-1} = T\left(1-\frac{2}{n}\right)$$

or $T' - T = \frac{2T}{n}$

$$= \frac{2\times24}{n} = \frac{48}{n}\text{hours.}$$

118. Six point masses of mass m each are at the vertices of a regular hexagon of side l. Calculate the force on any of the masses.

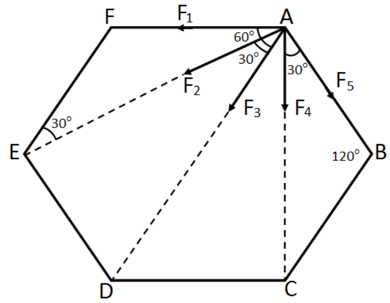
Ans. : Resultant force will be equal to sum of individual forces by each point mass (m). According to the diagram below, in which six point masses are placed at six vertices A, B, C, D, E and F.

$$AC = \sqrt{AB^2 + BC^2 - 2(AB)(BC)\cos 120^0}$$

$$= \sqrt{l^2 + l^2 - 2(l)(l)(-1/2)}$$

Similarly, $AE = l\sqrt{3}$

$$AD = \sqrt{AC^2 + CD^2} = \sqrt{3l^2 + l^2} = 2l$$



Force on mass m at A due to mass m at F is,

$$F_1 = \frac{Gm^2}{l^2} \text{ along AF}$$

Force on mass m at A due to mass m at E is,

$$F_2 = \frac{Gm^2}{(\sqrt{3}l)^2} = \frac{Gm^2}{3l^2} \text{ along AE } [\because AC = \sqrt{3}l]$$

Force on mass m at A due to mass m at D is,

$$F_3 = \frac{Gm\times m}{(2l)^2} = \frac{Gm^2}{4l^2} \text{ along AD } [\because AD = 2l]$$

Force on mass m at A due to mass m at C is,

$$F_4 = \frac{Gm\times m}{(\sqrt{3}l)^2} = \frac{Gm^2}{3l^2} \text{ along AC.}$$

Force on mass m at A due to mass m at B is,

$$F_5 = \frac{Gm\times m}{l^2} = \frac{Gm^2}{l^2} \text{ along AB}$$

Since F_1 and F_5 are equal in magnitude and make equal angle (i.e., 60° each) withAD, their resultant is along AD.

$$F_{15} = \sqrt{F_1^2 + F_5^2 + 2F_1F_5\cos 120^0}$$

$$= \frac{Gm^2}{l^2} \text{ along AD } [\because \text{Angle between } F_1 \text{ and } F_5 = 120^0]$$

Similarly, resultant force due to F_2 and F_4 is also along AD,

$$F_{24} = \sqrt{F_2^2 + F_4^2 + 2F_2F_4\cos 60^0}$$

$$= \frac{\sqrt{3}Gm^2}{3l^2} = \frac{GM^2}{\sqrt{3l^2}} \text{ along AD}$$

So, net force along AD

$$F_{net} = F_{15} + F_{24} + F_3$$

$$= \frac{GM^2}{l^2} + \frac{GM^2}{\sqrt{3l^2}} + \frac{GM^2}{4l^2}$$

$$= \frac{GM^2}{l^2}\left(1 + \frac{1}{\sqrt{3}} + \frac{1}{4}\right)$$

119. A satellite is in an elliptic orbit around the earth with aphelion of 6R and perihelion of 2R where R = 6400km is the radius of the earth. Find eccentricity of the orbit. Find the velocity of the satellite at apogee and perigee. What should be done if this satellite has to be transferred to a circular orbit of radius 6R? [G = 6.67 × 10^{-11} SI units and M = 6 × 10^{24}kg]

Ans. : r_p = 2R r_a =6R

Hence, r_p = a(1 - e) = 2R ...(i)

r_a = a(1 + e) = 6R ...(ii)

on dividing (i) by (ii)

$$\frac{1-e}{1+e} = \frac{2}{6}$$

$$3 - 3e = 1 + e$$

$$4e = 2 \Rightarrow e = \frac{1}{2}$$

There is not external force or torque on system.

So by the law of conservation of angular momentum.

$$L_1 = L_2$$

$m_a v_a r_a = m_p v_p r_p$ $m_a = m_p = m$ = mass of satellite

$$\therefore \frac{v_a}{v_p} = \frac{r_p}{r_a}\frac{2R}{6R} = \frac{1}{3}$$

So, $v_p = 3v_a$

Apply conservation of energy at apogee an d perigee ...(iii)

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a}$$

Multiplying $\frac{2}{m}$ to both side and putting r_p = 2R and r_a = 6R

$v_p^2 - \frac{2GM}{2R} = v_a^2 - \frac{2GM}{6R}$ (where M is mass of earth)

$$v_a = \frac{v_p}{3}$$

$$v_p^2 - v_a^2 = \frac{GM}{R} - \frac{1}{3}\frac{GM}{R}$$

$$v_p^2 - \left(\frac{v_p}{3}\right)^2 = \frac{GM}{R}\left[1 + \frac{1}{3}\right]$$

$$v_p^2\frac{8}{9} = \frac{GM}{R}\cdot\frac{2}{3}$$

$$v_p^2 = \frac{GM}{R}\frac{2}{3} \times \frac{9}{8} = \frac{3}{4}\frac{GM}{R}$$

$$v_p = \sqrt{\frac{3}{4}\frac{GM}{R}} = \sqrt{\frac{3 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{4 \times 6.6 \times 10^{6}}}$$

$$= \sqrt{\frac{9 \times 667 \times 10^{24-6-11-1}}{128}}$$

$$v_p = \sqrt{\frac{6003 \times 10^{18-11-1}}{128}} = \sqrt{46.89 \times 10^{6}}$$

$$= 6.85 \times 10^{3} m/s = 6.85 km/s$$

$$v_a = \frac{v_p}{3} = \frac{6.85}{3} = 2.28 km/s$$

$$v_c = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{6R}}$$

$$= \sqrt{\frac{6.67 \times 6 \times 10^{24-11}}{6 \times 6.8 \times 10^{6}}} = \sqrt{\frac{667}{640} \times 10^{13-6}}$$

$$= \sqrt{1.042 \times 10 \times 10^{6}} = \sqrt{10.42 \times 10^{6}}$$

$$v_c = 3.23 km/s$$

Hence to transfer to a circular orbit at apogee we have to boost the velocity by

$$v_0 - v_a = (3.23 - 2.28) = 0.95 km/\,s$$

120. Earth's orbit is an ellipse with eccentricity 0.0167. Thus, earth's distance from the sun and speed as it moves around the sun varies from day to day. This means that the length of the solar day is not constant through the year. Assume that earth's spin axis is normal to its orbital plane and find out the length of the shortest and the longest day. A day should be taken from noon to noon. Does this explain variation of length of the day during the year?

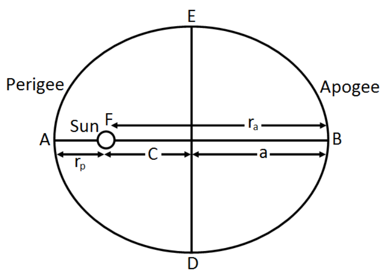
Ans. :

According to the diagram,

r_p = radius of perigee = 2R

r_a = radius of apogee = 6R

a = semi - major axis of the ellipse



Hence, we can write

r_a = a(1 + e) = 6R

P_p = a(1 - e) = 2R

$$\frac{a(1+e)}{a(1-e)} = \frac{6R}{2R} = 3$$

By solving, we get eccentricity $e = \frac{1}{2}$

If v_a and v_p are the velocities of the satellite (of mass m) at aphelion and perihelion respectively, then by conservation of angular momentum

$$\therefore\ L_{at\ perigee} = L_{at\ apogee}$$

$$mv_pr_p = mv_ar_a$$

$$\therefore \ \frac{v_a}{v_p} = \frac{r_p}{r_a} = \frac{1}{3}$$

Applying conservation of energy,

Energy at perigee = Energy at apogee

where M is the mass of the earth

$$\therefore \ v_p^2\left(1-\frac{1}{9}\right) = -2GM\left(\frac{1}{r_a}-\frac{1}{r_p}\right)$$

$$= 2GM\left(\frac{1}{r_p}-\frac{1}{r_a}\right) \text{ (By putting } v_a = \frac{v_p}{3}\text{)}$$

$$v_p = \frac{\left[2GM\left(\frac{1}{r_p}-\frac{1}{r_a}\right)\right]^{\frac{1}{2}}}{\left[1-\left(\frac{v_a}{v_p}\right)^2\right]^{\frac{1}{2}}}$$

$$= \left[\frac{\frac{2GM}{R}\left(\frac{1}{2}-\frac{1}{6}\right)}{\left(1-\frac{1}{9}\right)}\right]^{\frac{1}{2}} = \left(\frac{\frac{2}{3}}{\frac{8}{9}}\frac{GM}{R}\right)^{\frac{1}{2}} = \sqrt{\frac{3}{4}\frac{GM}{R}} = 6.85\text{km/ s}$$

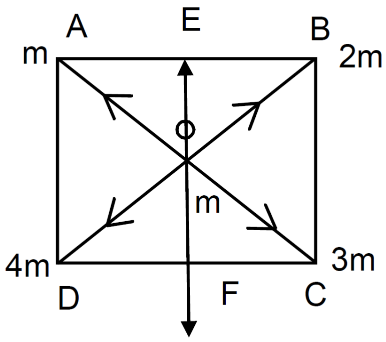
$$v_p = 6.85\text{km/ s}, \ v_a = 2.28\text{km/ s},$$

For circular orbit of radius r,

v_c = orbital velocity $= \sqrt{\frac{GM}{r}}$

For r = 6R, $v_c = \sqrt{\frac{GM}{6R}}$ = 3.23km/ s

121. Four particles having masses m, 2m, 3m and 4m are placed at the four corners of a square of edge a. Find the gravitational force acting on a particle of mass m placed at the centre.



Ans. :

To calculate the gravitational force on 'm' at unline due to other mouse.

$$\overrightarrow{F_{OD}} = \frac{G\times m\times 4m}{\left(\frac{a}{r^2}\right)^2} = \frac{8Gm^2}{a^2}$$

$$\overrightarrow{F_{OI}} = \frac{G\times m\times 2m}{\left(\frac{a}{r^2}\right)^2} = \frac{6Gm^2}{a^2}$$

$$\overrightarrow{F_{OB}} = \frac{G\times m\times 2m}{\left(\frac{a}{r^2}\right)^2} = \frac{4Gm^2}{a^2}$$

$$\overrightarrow{F_{OA}} = \frac{G\times m\times 2m}{\left(\frac{a}{r^2}\right)^2} = \frac{2Gm^2}{a^2}$$

Resultant $\overrightarrow{F_{OF}} = \sqrt{64\left(\frac{Gm^2}{a^2}\right)^2 + 36\left(\frac{Gm^2}{a^2}\right)^2} = 10\frac{Gm^2}{a^2}$

Resultant $\overrightarrow{F_{OE}} = \sqrt{64\left(\frac{Gm^2}{a^2}\right)^2 + 4\left(\frac{Gm^2}{a^2}\right)^2} = 2\sqrt{5}\frac{Gm^2}{a^2}$

The net resultant force will be,

$$F = \sqrt{100\left(\frac{Gm^2}{a^2}\right)^2 + 20\left(\frac{Gm}{a^2}\right)^2 - 2\left(\frac{Gm^2}{a^2}\right) \times 20\sqrt{5}}$$

$$= \sqrt{\left(\frac{Gm^2}{a^2}\right)^2 (120 - 40\sqrt{5})}$$

$$= \sqrt{\left(\frac{Gm^2}{a^2}\right)^2 (120 - 89.6)}$$

$$= \frac{Gm^2}{a^2}\sqrt{40.4} = 4\sqrt{2}\frac{Gm^2}{a^2}$$

122.

a. Find the radius of the circular orbit of a satellite moving with an angular speed equal to the angular speed of earth's rotation.

b. If the satellite is directly above the north pole at some instant, find the time it takes to come over the equatorial plane. Mass of the earth = 6 × 10^{24}kg.

Ans. : Angular speed f earth & the satellite will be same

$\frac{2\pi}{T_e} = \frac{2\pi}{T_s}$ or $\frac{1}{24\times3600} = \frac{1}{2\pi\sqrt{\frac{(R+h)^3}{gR^2}}}$

$$3600 = 3.14\sqrt{\frac{(R+h)^3}{gR^2}}$$

$$\frac{(R+h)^2}{gR^2} = \frac{(12\times3600)^2}{(3.14)^2}$$

$$\frac{(6400+h)^3\times10^9}{9.8\times(6400)^2\times10^6} = \frac{(12\times3600)^2}{(3.14)^2}$$

$$\frac{(6400+h)^3\times10^9}{6272\times10^9} = 432 \times 10^4$$

$$(6400 + h)^3 = 6272 \times 432 \times 10^4$$

$$6400 + h = \left(6272 \times 432 \times 10^4\right)^{\frac{1}{3}}$$

$$h = \left(6272 \times 432 \times 10^4\right)^{\frac{1}{3}} - 6400$$

$$= 42300cm$$

Time taken from north pole to equator $= \frac{1}{2}t$

$$= \left(\frac{1}{2}\right) \times 6.28\sqrt{\frac{(43200+6400)^3}{10\times(6400)^2\times10^6}}$$

$$= 3.14\sqrt{\frac{(497)^3\times10^6}{(64)^2\times10^{11}}}$$

$$= 3.14\sqrt{\frac{497\times497\times497}{64\times64\times10^5}}$$

$$= 6\ hour.$$

123. A satellite of mass 1000kg is supposed to orbit the earth at a height of 2000km above the earth's surface. Find

a. Its speed in the orbit.
b. Its kinetic energy.
c. The potential energy of the earth-satellite system.
d. Its time period. Mass of the earth = 6 × 10^{24}kg.

Ans. :

a. $\mathrm{V} = \sqrt{\frac{\mathrm{GM}}{\mathrm{r+h}}} = \sqrt{\frac{\mathrm{gr}^2}{\mathrm{r+h}}}$

$= \sqrt{\frac{9.8\times(6400\times10^3)^2}{10^6\times(6.4+2)}} = 6.9\times10^3\mathrm{m/s} = 6.9\mathrm{km/s}$

b. $\mathrm{K.E} = \left(\frac{1}{2}\right)\mathrm{mv}^2$

$= \left(\frac{1}{2}\right)1000\times(47.6\times10^6) = 2.38\times10^{10}\mathrm{J}$

c. $\mathrm{P.E} = \frac{\mathrm{GMm}}{-(\mathrm{R+h})}$

$= -\frac{6.67\times10^{-11}\times6\times10^{24}\times10^3}{(6400+2000)\times10^3}$

$= -\frac{40\times10^{13}}{8400} = -4.76\times10^{10}\mathrm{J}$

d. $\mathrm{T} = \frac{2\pi(\mathrm{r+h})}{\mathrm{V}} = \frac{2\times3.14\times8400\times10^3}{6.9\times10^3}$

$= 76.6\times10^2\,\mathrm{sec} = 2.1\text{ hour}$

124. Derive an expression for the gravitational field due to a uniform rod of length L and mass M at a point on its perpendicular bisector at a distance d from the centre.

Ans. : A small section of rod is considered at ‘x’ distance mass of the element = (M/L). dx = dm

$\mathrm{dE}_1 = \frac{\mathrm{G(dm)}\times1}{(\mathrm{d}^2+\mathrm{x}^2)} = \mathrm{dE}_2$

Resultant $\mathrm{dE} = 2\mathrm{dE}_1\sin\theta$

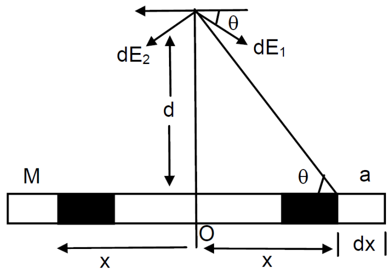
$= 2\times\frac{\mathrm{G(dm)}}{(\mathrm{d}^2+\mathrm{x}^2)}\times\frac{\mathrm{d}}{\sqrt{(\mathrm{d}^2+\mathrm{x}^2)}} = \frac{2\times\mathrm{GM}\times\mathrm{d\,dx}}{\mathrm{L}(\mathrm{d}^2+\mathrm{x}^2)\left(\sqrt{\mathrm{d}^2+\mathrm{x}^2}\right)}$

Total gravitational field

$\mathrm{E} = \int_0^{\frac{\mathrm{L}}{2}} \frac{2\mathrm{Gmd\,dx}}{\mathrm{L}\left(\sqrt{\mathrm{d}^2+\mathrm{x}^2}\right)^{\frac{3}{2}}}$

Integrating the above equation it can be found that,

$\mathrm{E} = \frac{2\mathrm{GM}}{\mathrm{d}\sqrt{\mathrm{L}^2+4\mathrm{d}^2}}$



125. A particle is fired vertically upward with a speed of 15km/s. With what speed will it move in intersteller space. Assume only earth's gravitational field.

Ans. : Initial velocity of the particle = 15km/s

Let its speed be 'v' at interstellar space.

$$\therefore \left(\frac{1}{2}\right)m[(15\times10^3)^2 - v^2] = \int_R^\infty \frac{GMm}{x^2}dx$$

$$\Rightarrow \left(\frac{1}{2}\right)m[(15\times10^3)^2 - v^2] = GMm\left[-\frac{1}{x}\right]_R^\infty$$

$$\Rightarrow \left(\frac{1}{2}\right)m[(225\times10^6) - v^2] = \frac{GMm}{R}$$

$$\Rightarrow 225\times10^6 - v^2 = \frac{2\times6.67\times10^{-11}\times6\times10^{24}}{6400\times10^3}$$

$$\Rightarrow v^2 = 225\times10^6 - \frac{40.02}{32}\times10^8$$

$$\Rightarrow v^2 = 225\times10^6 - 1.2\times10^8 = 10^8(1.05)$$

$$v = 1.01\times10^4 m/s$$

$$= 10 km/s$$

* Case study based questions [28]

126. Read the passage given below and answer the following questions from 1 to 5.

Satellites in a circular orbits around the earth in the equatorial plane with T = 24 hours are called Geostationary Satellites. Clearly, since the earth rotates with the same period, the satellite would appear fixed from any point on earth. It takes very powerful rockets to throw up a satellite to such large heights above the earth but this has been done in view of the several benefits of many practical applications. Thus radio waves broadcast from an antenna can be received at points far away where the direct wave fails to reach on account of the curvature of the earth. Waves used in television broadcast or other forms of communication have much higher frequencies and thus cannot be received beyond the line of sight. A Geostationery satellite, appearing fixed above the broadcasting station can however receive these signals and broadcast them back to a wide area on earth. The INSAT group of satellites sent up by India is one such group of geostationary satellites widely used for telecommunications in India. Another class of satellites is called the Polar satellites. These are low altitude (500 to 800 km) satellites, but they go around the poles of the earth in a north-south direction whereas the earth rotates around its axis in an east-west direction. Since its time period is around 100 minutes it crosses any altitude many times a day. However, since its height h above the earth is about 500-800 km, a camera fixed on it can view only small strips of the earth in one orbit. Adjacent strips are viewed in the next orbit, so that in effect the whole earth can be viewed strip by strip during the entire day. These satellites can view polar and equatorial regions. at close distances with good resolution. Information gathered from such satellites is extremely useful for remote sensing, meterology as well as for environmental studies of the earth.

i. Time period of geospatial satellite is:
   a. 24 hours
   b. 48 hours
   c. 72 hours
   d. None of these
ii. Polar satellites are approximately revolving at height of
   a. 500 to 800km
   b. 1500 to 2000km
   c. 3000 to 4000km
   d. None of these
iii. Which satellite used to view polar and equatorial regions?
iv. Write note on polar satellites
v. Write a note on geostationary satellite. Give its applications.

Ans. :

i. (a) 24 hours
ii. 500 to 800km
iii. Polar satellites are used to view polar and equatorial regions as they rotate on poles of earth.
iv. Polar satellites are low altitude (500 to 800 km) satellites, but they go around the poles of the earth in a north-south direction. Since its time period is around 100 minutes it crosses any altitude many times a day. Information gathered from such satellites is extremely useful for remote sensing, meterology as well as for environmental studies of the earth.
v. Satellites in circular orbits around the earth in the equatorial plane with time period same as earth are called Geostationary Satellites.

Applications:- Radio waves broadcast. Satellites widely used for telecommunications in India. GPS system, navigation system , defence etc.

127. Read the passage given below and answer the following questions from 1 to 5. If a stone is thrown by hand, we see it falls back to the earth. Of course using machines we can shoot an object with much greater speeds and with greater and greater initial speed, the object scales higher and higher heights. A natural query that arises in our mind is the following: can we throw an object with such high initial speeds that it does not fall back to the earth ? Thus minimum speed required to throw object to infinity away from earth's gravitational field is called escape velocity. $\mathrm{V_e} = \sqrt{(2\mathrm{gr})}$ Where g is acceleration due to gravity and r is radius of earth and after solving ve 11.2 km/s. This is called the escape speed, sometimes loosely called the escape velocity. This applies equally well to an object thrown from the surface of the moon with g replaced by the acceleration due to Moon's gravity on its surface and r replaced by the radius of the moon. Both are smaller than their values on earth and the escape speed for the moon turns out to be 2.3 km/s, about five times smaller. This is the reason that moon has no atmosphere. Gas molecules if formed on the surface of the moon having velocities larger than this will escape the gravitational pull of the moon. Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of

planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them. In particular, their orbits around the earth are circular or elliptic. Moon is the only natural satellite of the earth with a near circular orbit with a time period of approximately 27.3 days which is also roughly equal to the rotational period of the moon about its own axis.

i. Time period of moon is:
   a. 27.3 days
   b. 20 days
   c. 85 days
   d. None of these

ii. Escape velocity from earth is given by:
   a. 20 km/s
   b. 11.2 km/s
   c. 2 km/s
   d. None of these

iii. Define escape velocity. Give its formula.

iv. Why moon don't Have any atmosphere?

v. What is satellite? Which law governs them?

Ans. :

i. (a) 27.3 days

ii. (b) 11.2 km/s

iii. Minimum speed required to throw object to infinity away from earth's gravitational field is called escape velocity.

$$V_e = \sqrt{(2gr)}$$

Where g is acceleration due to gravity and r is radius of earth and after solving v_e 11.2 km/s. This is called the escape speed, sometimes loosely called the escape velocity.

iv. The escape speed for the moon turns out to be 2.3 km/s, about five times smaller than that of earth. Therefore all atmospheric gas can go easily out of atmosphere of moon. This is the reason that moon has no atmosphere.

v. Earth satellites are objects which revolve around the earth. Their motion is very similar to the motion of planets around the Sun and hence Kepler's laws of planetary motion are equally applicable to them.

128. Read the passage given below and answer the following questions from 1 to 5.

Acceleration due to gravity The acceleration for any object moving under the sole influence of gravity is known as acceleration due to gravity. So, for an object of mass m, the acceleration experienced by it is usually denoted by the symbol g which is related to F by Newton's second law by relation F = mg Thus, $g = \frac{F}{m} = \frac{Gm_e}{R_e^2}$ Acceleration g is readily measurable as Re is a known quantity. The measurement ofG by Cavendish's experiment (or otherwise), combined with knowledge of g and Re enables one to estimate M_e from the above equation. This is the reason why there is a popular statement regarding Cavendish "Cavendish weighed the earth". The value of g decrease as we go upwards from the earth's surface or downwards, but it is maximum at its surface.

i. If g is the acceleration due to gravity at the surface of the earth, the force acting on the particle of mass m placed at the surface is:

a. mg

b. $\frac{\mathrm{GmM}\theta}{\mathrm{R_e^2}}$

c. Data insufficient

d. Both (a) and (b)

ii. The weight of a body at the centre of earth is:

a. same as on the surface of earth

b. same as on the poles

c. same as on the equator

d. None of the above

iii. If the mass of the sun is ten times smaller and gravitational constant G is ten times larger in magnitude, then for earth:

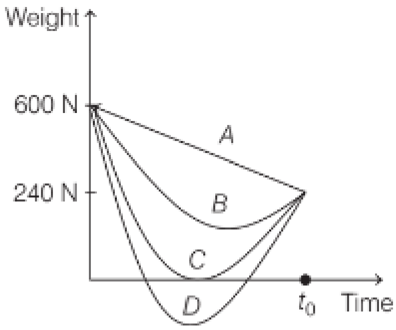
a. walking on ground would become more easy

b. acceleration due to gravity on the earth will not change

c. raindrops will fall much slower

d. airplanes will have to travel much faster

iv. Suppose, the acceleration due to gravity at the earth's surface is 10 ms $^{-2}$ and at the surface of mars, it is 4.0 ms $^{-2}$ 60kg passenger goes from the earth to the mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky. Which curve best represents the weight (net gravitational force) of the passenger as a function of time?



a. A

b. B

c. C

d. D

v. If the mass of the earth is doubled and its radius halved, then new acceleration due to the gravity $\acute{g}$ is:

a. $\acute{g} = 4g$

b. $\acute{g} = 8g$

c. $\acute{g} = g$

d. $\acute{g} = 16g$

Ans. :

i. (d) Both (a) and (b)

Explanation:

The force acting on the particle of mass m at surface of the earth,

F = mg ...(i)

where, g = acceleration due to gravity at the earth's surface.

Also, $g = \frac{GM_e}{R_e^2}$ ...(ii)

Then, from Eqs. (i) and (ii), we get

$\Rightarrow F = mg = \frac{GmM_e}{R_e^2}$

ii. (d) None of the above

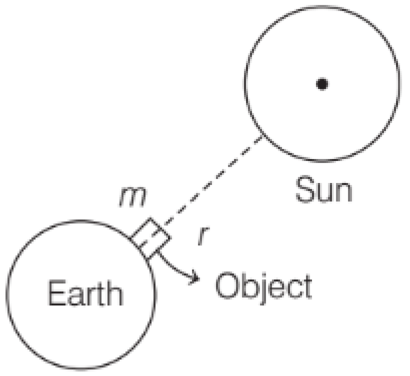
Explanation:

Gravitational acceleration (g) at the centre of earth is zero, hence weight of body (w = mg) at the centre of earth becomes zero.

iii. (d) airplanes will have to travel much faster

Explanation:

Consider the given diagram



Force on the object due to the earth,

$F = \frac{G\acute{M}_e m}{R^2} = \frac{10GM_eM}{R^2} (\because \acute{g} = 10G)$

$= 10\left(\frac{Gm_e m}{R^2}\right) = (10g)m = 10\ mg$ ...(i)

$\because g = \frac{GM_e}{R^2}$

Now, force on the object due to the sun,

$\acute{F} = \frac{G\acute{M}_s m}{r^2}$

$= \frac{G(M_s)m}{10\ r^2}$

$\left(\because \acute{M}_s = \frac{M_s}{10}\right)$

As, r R >> (radius of the earth)

⇒ F will be very small, so the effect of the sun will be neglected.

Now, as $\acute{g} = 10\ g$

Hence, weight of person $= m\acute{g} = 10\ mg$

$\left[_{\text{from Eq.(i)}}\right]$

i.e. Gravity pull on the person will increase. Due to it, walking on ground would become more difficult. Escape velocity v_e is proportional to g, i.e.

$V_e \infty\ g.$

As, $\acute{g} > g \Rightarrow v_e \acute{>} v_e$

Hence, rain drops will fall much faster. To overcome the increased gravitational force of the earth, the airplanes will have to travel much faster.

iv. (c)C

Explanation:

Initially, the weight of the passenger at the earth’s surface, w = mg = 60 × 10 = 600 N. Finally, the weight of the passenger at the surface of the mars = 60 × 4 = 240 N and during the flight in between somewhere its weight will be zero because at that point, gravitational pull of earth and mars will be equal.

v. (b) $\acute{\mathrm{g}} = 8\ \mathrm{g}$

Explanation:

As we know that, acceleration due to gravity,

$\mathrm{g} = \frac{\mathrm{Gm}}{\mathrm{R}^2}$

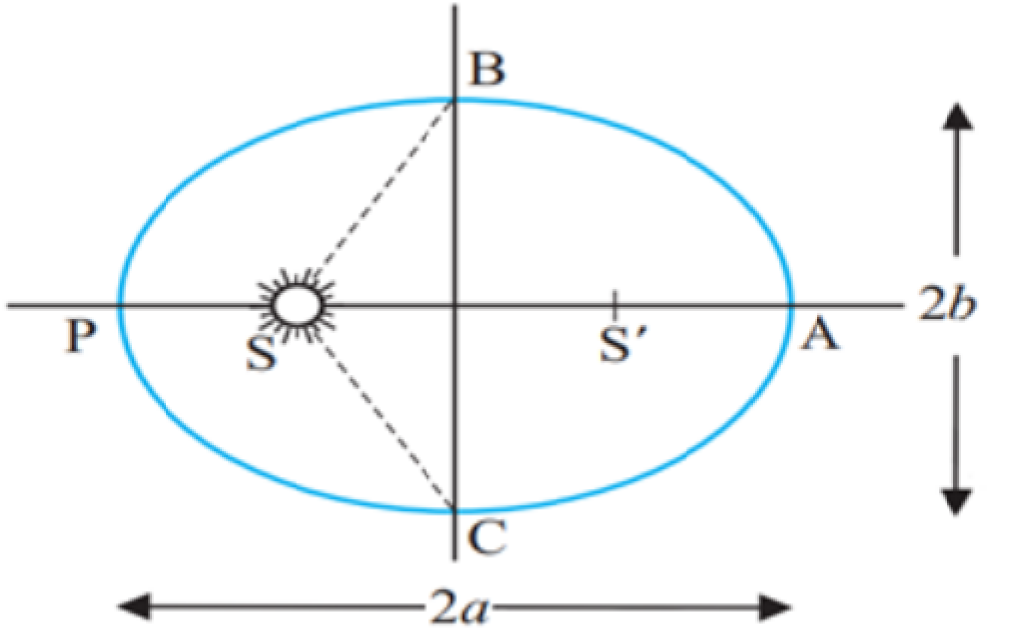
Given, $\acute{\mathrm{M}} = 2\mathrm{M}(\because \text{Mass gets doubled}))$

$\Rightarrow \acute{\mathrm{g}} = \frac{\mathrm{G}\acute{\mathrm{M}}}{\acute{\mathrm{R}}^2} = \frac{\mathrm{G}(2\mathrm{M})}{(\frac{\mathrm{R}}{2})^2} = \frac{8\ \mathrm{Gm}}{\mathrm{R}^2}$

$\therefore \acute{\mathrm{g}} = 8\ \mathrm{g}$

Thus, the new acceleration due to gravity $\acute{\mathrm{g}}$ is 8 times that of g.

129. Read the passage given below and answer the following questions from 1 to 5. LAW OF ORBIT: The orbit of every planet is an ellipse around the sun with sun at one of the two



foci of ellipse.
LAW OF AREAS: The line that joins a planet to the sun sweeps out equal areas in equal intervals of time. Area covered by the planet while revolving around the sun will be equal in equal intervals of time. This means the rate of change of area with time is constant. LAW OF PERIOD: According to this law the square of time period of a planet is directly proportional to the cube of the semi-major axis of its orbit. Suppose earth is revolving around the sun then the square of the time period (time taken to complete one revolution around sun) is directly proportional to the cube of the semi major axis. It is known as Law of Periods as it is dependent on the time period of planets. Answer the following.

i. Keplers second law is knows as:
   a. Law of period
   b. Law of area
   c. Law of gravity
   d. None of these
ii. Keplers third law is knows as:
   a. Law of period
   b. Law of area
   c. Law of gravity
   d. None of these
iii. The velocity of a planet is constant throughout its elliptical trajectory in an orbit.
   a. True
   b. False
   c. None of these
iv. State Kepler's second law of planetary motion.
v. Two objects of masses 5kg and 10 kg separated by distance 10m. What is gravitational force between them?

Ans. :

i. (c) Law of gravity
ii. (a) Law of period
iii. (a) True
iv. Keplers 3 laws are stated below.

LAW OF ORBIT: The orbit of every planet is an ellipse around the sun with sun at one of the two foci of ellipse.

LAW OF AREAS: The line that joins a planet to the sun sweeps out equal areas in equal intervals of time. Area covered by the planet while revolving around the sun will be equal in equal intervals of time. This means the rate of change of area with time is constant.

LAW OF PERIOD: According to this law the square of time period of a directly proportional to the cube of the semi-major axis of its orbit.

The force of attraction between any two unit masses separated by a unit distance is called universal gravitational constant denoted by G measured in Nm^2/kg^2.

v. Mathematically,

$$\mathrm{F} = \mathrm{G}\frac{\mathrm{m1}\times\mathrm{m2}}{\mathrm{d2}}$$

Here M1 = 5kg

M2 = 10kg

D = 10m

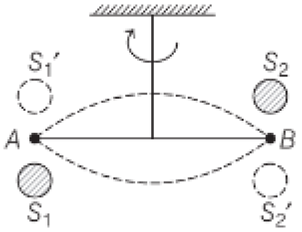
Then, forec is given by

$$\mathrm{F} = 6.67\times10^{-11}\times5\times\frac{10}{100}$$

F = 3.33×10^{-11}N.

130. Read the passage given below and answer the following questions from 1 to 5.

Cavendish's Experiment The figure shows the schematic drawing of Cavendish's experiment to determine the value of the gravitational constant. The bar AB has two small lead spheres attached at its ends. The bar is suspended from a rigid support by a fine wire. Two large lead spheres are brought close to the small ones but on opposite sides as shown. The value of G from this experiment came to be $6.67 \times 10^{-11} \frac{\text{N-m}^2}{\text{Kg}^2}$



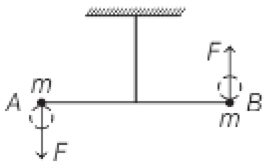
i. The big spheres attract the nearby small ones by a force which is:
   a. equal and opposite
   b. equal but in same direction
   c. unequal and opposite
   d. None of the above
ii. The net force on the bar is:
   a. non-zero
   b. zero
   c. Data insufficient
   d. None of these
iii. The net torque on the bar is:
   a. zero
   b. non-zero
   c. F times the length of the bar, where F is the force of attraction between a big sphere and its neighbouring
   d. Both (b) and (c)
iv. The torque produces twist in the suspended wire. The twisting stops when:
   a. restoring torque of the wire equals the gravitational torque
   b. restoring torque of the wire exceeds the gravitational torque
   c. the gravitational torque exceeds the restoring torque of the wire
   d. None of the above
v. After Cavendish's experiment, there have been given suggestions that the value of the gravitational constant G becomes smaller when considered over very large time period (in billions of years) in the future. If that happens, for our earth:
   a. nothing will change
   b. we will become hotter after billions of years
   c. we will be going around but not strictly in closed orbits
   d. None of the above

Ans. :

i. (a) equal and opposite.

Explanation:

The force of attraction on small spheres due to big sphere are equal and opposite in direction. Hence, equal and opposite force separated by a fixed distance forms a couple.



ii. (b) zero

Explanation:

$$\left|F_{net}\right| = \text{Zero}$$

iii. (d) Both (b) and (c)

Explanation:

Magnitude of torque due to a couple

= (Either Force) × (Distance between of forces)

= F × l

where, l = length of the bar and F = force of attraction between a big sphere and its neighbouring small sphere.

iv. (a) restoring torque of the wire equals the gravitational torque.

Explanation:

The torque produces a twist in the suspended wire. The twisting stops when the restoring torque of the wire equal the gravitational torque.

v. (c) we will be going around but not strictly in closed orbits

Explanation:

We know that, gravitational force between the earth and the sun.

$F_g = \frac{GMm}{r^2}$,

where M is mass of the sun and m is mass of the earth. When G decreases with time, the gravitational force F_G will become weaker with time. As F_G is changing with time. Due to it, the earth will be going around the sun not strictly in closed orbit and radius also increases, since the attraction force is getting weaker. Hence, after long time the earth will leave the solar system.

131. Find the acceleration due to gravity of the moon at a point 1000km above the moon's surface. The mass of the moon is 7.4 × 10^{22}kg and its radius is 1740km.

Ans. : $\frac{GM}{(R+h)^2} = \frac{6.67\times10^{-11}\times7.4\times10^{22}}{(1740+1000)^2\times10^6}$

$= \frac{49.358\times10^{11}}{2740\times2740\times10^6}$

$= \frac{49.358\times10^{11}}{0.75\times10^{13}}$

$= 65.8 \times 10^{-2} = 0.65 m/s^2$

132. No part of India is situated on the equator. Is it possible to have a geostationary satellite which always remains over New Delhi?

Ans. : No, all geostationary orbits are concentric with the equator of the Earth.

----- one minute can't change your life but utilisation of every minute definitely change your life -----